4.5) Linear transformations of roots

Worked example

The quadratic equation $x^2 + 3x - 10 = 0$ has roots α and β.
Without finding the roots, determine the equation with roots $\alpha - 2$ and $\beta - 2$

Your turn

The quadratic equation $x^2 - 3x - 10 = 0$ has roots α and β.
Without finding the roots, determine the equation with roots $\alpha - 1$ and $\beta - 1$

$w^2 - w - 12 = 0$

Worked example

The cubic equation $x^3 + 2x^2 - 3x + 4 = 0$ has roots α, β and γ. Find the equation of the polynomial with roots:

$3\alpha, 3\beta$ and 3γ

Your turn

The cubic equation $x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β and γ. Find the equation of the polynomial with roots:

$2\alpha, 2\beta$ and 2γ

$w^3 - 4w^2 + 12w - 32 = 0$

Worked example

The cubic equation $x^3 + 2x^2 - 3x + 4 = 0$ has roots α, β and γ. Find the equation of the polynomial with roots:

$$\alpha - 2, \beta - 2 \text{ and } \gamma - 2$$

Your turn

The cubic equation $x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β and γ. Find the equation of the polynomial with roots:

$$\alpha + 3, \beta + 3 \text{ and } \gamma + 3$$

$$w^3 - 11w^2 + 42w - 58 = 0$$

Worked example

The cubic equation $x^3 + 2x^2 - 4 = 0$ has roots $\alpha, \beta, \gamma, \delta$. Find the equation with roots $(2\alpha + 1)$, $(2\beta + 1)$, $(2\gamma + 1)$ and $(2\delta + 1)$

Your turn

The cubic equation $x^3 - 2x^2 + 4 = 0$ has roots $\alpha, \beta, \gamma, \delta$. Find the equation with roots $(3\alpha - 1)$, $(3\beta - 1)$, $(3\gamma - 1)$ and $(3\delta - 1)$

$$w^3 - 3w^2 - 9w + 103 = 0$$

Worked example

The quartic equation $x^4 + 3x^3 - x^2 - 15x - 1 = 0$ has roots α, β, γ and δ. Find the equation of the polynomial with roots:

$(3\alpha - 1)$, $(3\beta - 1)$, $(3\gamma - 1)$ and $(3\delta - 1)$

Your turn

The quartic equation $x^4 - 3x^3 + 15x + 1 = 0$ has roots α, β, γ and δ. Find the equation of the polynomial with roots:

$(2\alpha + 1)$, $(2\beta + 1)$, $(2\gamma + 1)$ and $(2\delta + 1)$

$w^4 - 10w^3 + 24w^2 + 98w - 97 = 0$