

Expressions Related to the Roots of Polynomials

Sums of squares:

- Quadratic: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
- Cubic: $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$
- Quartic: $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2(\alpha\beta + \alpha\beta + \alpha\gamma + \beta\gamma + \dots)$

Sums of cubes:

- Quadratic: $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
- Cubic: $\alpha^3 + \beta^3 + \gamma^3 = (\alpha + \beta + \gamma)^3 - 3(\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha) + 3\alpha\beta\gamma$

Reciprocals:

- Quadratic: $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$
- Cubic: $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$
- Quartic: $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{\Sigma\alpha\beta\gamma}{\alpha\beta\gamma\delta}$

Products of Powers

- Quadratic: $\alpha^n + \beta^n = (\alpha\beta)^n$
- Cubic: $(\alpha + \beta + \gamma)^n = (\alpha\beta\gamma)^n$
- Quartic: $(\alpha + \beta + \gamma + \delta)^n = (\alpha\beta\gamma\delta)^n$

Example

The three roots of a cubic equation are α , β and γ . Given that $\alpha\beta\gamma = 4$, $\alpha\beta + \beta\gamma + \gamma\alpha = -5$ and $\alpha + \beta + \gamma = 3$, find the value of

$$(\alpha + 3)(\beta + 3)(\gamma + 3)$$