Roots of Quartics

Polynomial	Sum of roots	Sum of possible products of pairs of roots	Sum of products of triples	Sum of products of quadruples
Quadratic $ax^2 + bx + c$ (Roots: α, β)	$\alpha + \beta = -\frac{b}{a}$	$\alpha\beta = \frac{c}{a}$	N/A	N/A
Cubic $ax^3 + bx^2 + cx + d$ (Roots: α, β, γ)	$\alpha + \beta + \gamma = -\frac{b}{a}$	$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$	$\alpha\beta\gamma = -\frac{d}{a}$	N/A
Quartic $ax^4 + bx^3 + cx^2 + dx + e$ (Roots: $\alpha, \beta, \gamma, \delta$)	$\alpha + \cdots + \delta = -\frac{b}{a}$	$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$	$\alpha\beta\gamma + \alpha\beta\delta + \cdots = -\frac{d}{a}$	$\alpha\beta\gamma\delta = \frac{e}{a}$

Example

Find the quartic equation with roots 1, -2 and 3 (repeated).

Example

The equation $x^4 + 2x^3 + px^2 + qx - 60 = 0,\ x \in \mathbb{C},\ p, q \in \mathbb{R}$, has roots $\alpha,\ \beta, \gamma, \delta$. Given that $\gamma = -2 + 4i$ and $\delta = \gamma^*$.

(a) Show that $\alpha + \beta - 2 = 0$ and that $\alpha\beta + 3 = 0$

(b) Hence find all the roots of the quartic equation and find the values of p and q.

Ex 4C pg 60-61