4.3) Roots of a quartic equation

Worked example

α, β, γ, and δ are the roots of the quartic equation $x^4 - 3x^3 - 2x^2 + x - 4 = 0$

Find the values of:

(a) $\alpha + \beta + \gamma + \delta$

(b) $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

(c) $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

(d) $\alpha\beta\gamma\delta$

(e) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

(f) $\alpha^2\beta^2\gamma^2\delta^2$

Your turn

α, β, γ, and δ are the roots of the quartic equation $x^4 + 3x^3 + 2x^2 - x + 4 = 0$

Find the values of:

(a) $\alpha + \beta + \gamma + \delta$

(b) $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta$

(c) $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta$

(d) $\alpha\beta\gamma\delta$

(e) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}$

(f) $\alpha^3\beta^3\gamma^3\delta^3$

(a) -3

(b) 2

(c) 1

(d) 4

(e) $\frac{1}{4}$

(f) 64

Worked example

The roots of a quartic equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

are $\alpha = \frac{2}{3}, \beta = \frac{1}{2}, \gamma = 2$ and $\delta = -\frac{3}{2}$

Find integer values of a, b and c

Your turn

The roots of a quartic equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

are $\alpha = -\frac{3}{2}, \beta = -\frac{1}{2}, \gamma = -2$ and $\delta = \frac{2}{3}$

Find integer values of a, b and c

$$a = 12, b = 40, c = 25, d = -20, e = -12$$

Worked example

The equation $x^4 + 4x^3 + px^2 + qx - 80 = 0, x \in \mathbb{C}, p, q \in \mathbb{R}$, has roots $\alpha, \beta, \gamma, \delta$.
Given that $\gamma = -4 + 2i$ and $\delta = \gamma^*$.

(a) Show that $\alpha + \beta - 4 = 0$ and that $\alpha\beta + 4 = 0$

(b) Hence find all the roots of the quartic equation and find the values of p and q.

Your turn

The equation $x^4 + 2x^3 + px^2 + qx - 60 = 0, x \in \mathbb{C}, p, q \in \mathbb{R}$, has roots $\alpha, \beta, \gamma, \delta$.
Given that $\gamma = -2 + 4i$ and $\delta = \gamma^*$.

(a) Show that $\alpha + \beta - 2 = 0$ and that $\alpha\beta + 3 = 0$

(b) Hence find all the roots of the quartic equation and find the values of p and q.

(a) Shown

(b) $p = 9, q = -52$