Volumes of revolution for parametric curves

We have seen in Pure Year 2 that parametric equations are where, instead of some single equation relating x and y, we have an equation for each of x and y in terms of some parameter, e.g. t. As t varies, this generates different points (x, y).

To integrate parametrically, the trick was to replace dx with $\frac{dx}{dt}\, dt$

$$\boldsymbol{V = \pi \int_{x=b}^{x=a} y^2 \, dx}$$

Note that as we're integrating with respect to t now, we need to find the equivalent limits for t. We can do the same for revolving around the y-axis: just replace dy with $\frac{dy}{dt}$ and change the limits.

Example

The curve C has parametric equations $x = t(1+t)$, $y = \frac{1}{1+t}$, $t \geq 0$.

The region R is bounded by C, the x-axis and the lines $x = 0$ and $y = 0$. Find the exact volume of the solid formed when R is rotated 2π radians about the x-axis.

Test Your Understanding

Edexcel C4(Old) June 2011 Q7

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

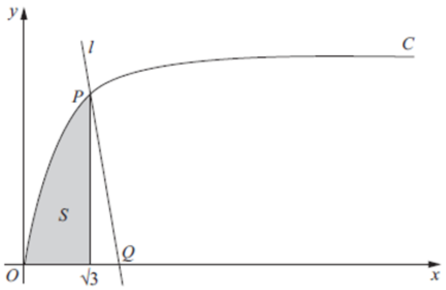

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta < \frac{\pi}{2}.$$

Ex4C p. 84-87