

4.3) Volumes of revolution of parametrically defined curves

Worked example

The curve C has parametric equations

$$x = t(1 - t), y = \frac{1}{1-t}, t \leq 0.$$

The region R is bounded by C , the x -axis and the lines $x = 0$ and $x = -2$.

Find the exact volume of the solid formed when R is rotated 2π radians about the x -axis.

Your turn

The curve C has parametric equations

$$x = t(1 + t), y = \frac{1}{1+t}, t \geq 0.$$

The region R is bounded by C , the x -axis and the lines $x = 0$ and $x = 2$.

Find the exact volume of the solid formed when R is rotated 2π radians about the x -axis.

$$\pi\left(2 \ln 2 - \frac{1}{2}\right)$$

Worked example

A curve C has parametric equations

$$x = \tan \theta, y = \sec^3 \theta, 0 \leq \theta < \frac{\pi}{2}$$

A finite region is bounded by the curve C, the y -axis, and the lines $y = 1$ and $y = 8$. Find the exact volume of the solid formed when this region is rotated 2π radians about the y -axis.

Your turn

A curve C has parametric equations

$$x = \tan \theta, y = \sin \theta, 0 \leq \theta < \frac{\pi}{2}$$

A finite region is bounded by the curve C, the line $x = \sqrt{3}$ and the x -axis. Find the exact volume of the solid formed when this region is rotated 2π radians about the x -axis.

$$\pi\sqrt{3} - \frac{1}{3}\pi^2$$

Worked example

A curve C has parametric equations

$$x = \frac{1}{3t}, y = \ln 3t, t \geq \frac{1}{3}$$

A finite region is bounded by the curve C, the x -axis, the y -axis and the line $y = a$.

Given that the volume of the solid formed when this region is rotated 2π radians about the y -axis is $\frac{12\pi}{25}$, find the exact value of a

Your turn

A curve C has parametric equations

$$x = \frac{1}{2t}, y = \ln 2t, t \geq \frac{1}{2}$$

A finite region is bounded by the curve C, the x -axis, the y -axis and the line $y = a$.

Given that the volume of the solid formed when this region is rotated 2π radians about the y -axis is $\frac{24\pi}{49}$, find the exact value of a

$$a = \ln 7$$

Worked example

A curve C has parametric equations

$$x = 2 \cos t, y = t^2, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$$

A finite region is bounded by the curve C and the y -axis. Find the exact volume of the solid formed when this region is rotated 2π radians about the y -axis

Your turn

A curve C has parametric equations

$$x = 2 \sin t, y = t^2, 0 \leq t \leq \pi$$

A finite region is bounded by the curve C and the y -axis. Find the exact volume of the solid formed when this region is rotated 2π radians about the y -axis

$$2\pi^3$$