

4.2) Volumes of revolution around the y -axis

Worked example

A finite region is bounded by the curve with equation $y = 8 \ln x - 1$, the x -axis, the y -axis, and the line $y = 2$

Find the exact volume of the solid formed when the region is rotated by 2π radians about the y -axis.

Your turn

A finite region is bounded by the curve with equation $y = 4 \ln x - 1$, the x -axis, the y -axis, and the line $y = 4$

Find the exact volume of the solid formed when the region is rotated by 2π radians about the y -axis.

$$2\pi\sqrt{e}(e^2 - 1)$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$x = e^y - e^{-2y} \text{ between } y = 0 \text{ and } y = 1$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$x = e^{2y} - e^{-y} \text{ between } y = 0 \text{ and } y = 1$$

$$\frac{\pi}{4}(e^2 - 1)$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$x = \frac{\sqrt{4-\ln y}}{y} \text{ between } y = 1 \text{ and } y = 4$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$x = \frac{\sqrt{5-\ln y}}{y} \text{ between } y = 1 \text{ and } y = 5$$

$$\frac{\pi}{5} (\ln 5 + 16)$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \frac{2}{x} - 2 \text{ between } y = 0 \text{ and } y = 1$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \frac{1}{x} - 1 \text{ between } y = 0 \text{ and } y = 1$$

$$\frac{\pi}{2}$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \frac{2-5x^2}{1-x^2} \text{ between } y = -1 \text{ and } y = 1$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \frac{5-2x^2}{x^2-1} \text{ between } y = -1 \text{ and } y = 1$$

$$\pi(2 + 3 \ln 3)$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = 3e^{x^2} \text{ between } y = 3 \text{ and } y = 6$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = 2e^{x^2} \text{ between } y = 2 \text{ and } y = 4$$

$$\pi(4 \ln 2 - 2)$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \arcsin \sqrt{x} \text{ between } y = 0 \text{ and } y = \frac{\pi}{2}$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the y -axis between the given limits:

$$y = \arccos \sqrt{x} \text{ between } y = 0 \text{ and } y = \frac{\pi}{2}$$

$$\frac{3\pi^2}{16}$$

Worked example

A finite region is bounded by the curve with equation $x = \frac{1}{3y+1}$, the y -axis and the lines $y = 1$ and $y = b$. The region is rotated through 2π radians about the y -axis to generate a solid of revolution. Given that the volume of this solid is $\frac{\pi}{60}$, find b

Your turn

A finite region is bounded by the curve with equation $x = \frac{1}{2y+1}$, the y -axis and the lines $y = 1$ and $y = b$. The region is rotated through 2π radians about the y -axis to generate a solid of revolution. Given that the volume of this solid is $\frac{\pi}{10}$, find b

$$b = \frac{13}{4}$$