4.2) Roots of a cubic equation

Worked example

α, β and γ are the roots of the cubic equation $2x^3 - 3x^2 + 4x - 2 = 0$.

Without solving the equation, find the values of:

(a) $\alpha + \beta + \gamma$ (b) $\alpha\beta + \beta\gamma + \gamma\alpha$ (c) $\alpha\beta\gamma$ (d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

Your turn

α, β and γ are the roots of the cubic equation $2x^3 + 3x^2 - 4x + 2 = 0$.

Without solving the equation, find the values of:

(a) $\alpha + \beta + \gamma$ (b) $\alpha\beta + \beta\gamma + \gamma\alpha$ (c) $\alpha\beta\gamma$ (d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

(a) $-\frac{3}{2}$

(b) -2

(c) -1

(d) 2

Worked example

α, β and γ are the roots of the cubic equation $2x^3 - 3x^2 + 4x - 6 = 0$.
Without solving the equation, find the values of $\alpha^2\beta^2\gamma^2$ and $\alpha^3\beta^3\gamma^3$

Your turn

α, β and γ are the roots of the cubic equation $2x^3 + 3x^2 - 4x + 4 = 0$.
Without solving the equation, find the values of $\alpha^2\beta^2\gamma^2$ and $\alpha^3\beta^3\gamma^3$

$\alpha^2\beta^2\gamma^2 = 4$ and $\alpha^3\beta^3\gamma^3 = -8$

Worked example

The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = 1 - 3i, \beta = 1 + 3i$ and $\gamma = 3$.
Find integers values for a, b, c and d.

Your turn

The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = 1 - 2i, \beta = 1 + 2i$ and $\gamma = 2$.
Find integers values for a, b, c and d.

$$a = 1, b = -4, c = 9, d = -10$$

Worked example

The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = \frac{2}{3}, \beta = \frac{1}{3}$ and $\gamma = 1$.

Find integers values for a, b, c and d.

Your turn

The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are $\alpha = \frac{3}{2}, \beta = \frac{1}{2}$ and $\gamma = 1$.

Find integers values for a, b, c and d.

$$a = 4, b = -12, c = 11, d = -3$$

Worked example

The cubic equation $x^3 - 42x^2 + 336x - 512 = 0$ has roots $\alpha, k\alpha$, and $k^2\alpha$ for some real constant k. Find the values of α and k

Your turn

The cubic equation $x^3 - 14x^2 + 56x - 64 = 0$ has roots $\alpha, k\alpha$, and $k^2\alpha$ for some real constant k. Find the values of α and k

$\alpha = 2, k = 2$ or $\alpha = 8, k = \frac{1}{2}$