

4.1) Volumes of revolution around the x -axis

Worked example

A finite region is bounded by the curve with equation $y = \sin 4x$, the x -axis and $x = \frac{\pi}{4}$. Find the volume of the solid formed when the region is rotated through 2π radians about the x -axis.

Your turn

A finite region is bounded by the curve with equation $y = \sin 2x$, the x -axis and $x = \frac{\pi}{2}$. Find the volume of the solid formed when the region is rotated through 2π radians about the x -axis.

$$\frac{\pi^2}{4}$$

Worked example

A finite region is bounded by the curve with equation $y = 1 - 2 \sin x$ ($0 < x < \pi$) and the x -axis.

Find the exact volume of the solid formed when the region is rotated through 2π radians about the x -axis.

Your turn

A finite region is bounded by the curve with equation $y = 1 - 2 \cos x$ ($0 < x < \pi$) and the x -axis.

Find the volume of the solid formed when the region is rotated through 2π radians about the x -axis.

$$\pi(4\pi + 3\sqrt{3})$$

Worked example

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the x -axis between the given limits:

$$y = \sqrt{\frac{3 \sin x}{2 + \cos x}} \text{ between } x = 0 \text{ and } x = \frac{\pi}{2}$$

Your turn

Find the exact volume of the solid generated when each curve is rotated through 2π radians about the x -axis between the given limits:

$$y = \sqrt{\frac{4 \sin x}{1 + \cos x}} \text{ between } x = 0 \text{ and } x = \frac{\pi}{2}$$

$$4\pi \ln 2$$

Worked example

Using integration by parts, find the exact volume of the solid generated when each curve is rotated through 2π radians about the x -axis between the given limits:

$$y = \frac{\sqrt{\ln x}}{x^2} \text{ between } x = 1 \text{ and } x = 2$$

Your turn

Using integration by parts, find the exact volume of the solid generated when each curve is rotated through 2π radians about the x -axis between the given limits:

$$y = \sqrt{x} \sec x \text{ between } x = 0 \text{ and } x = \frac{\pi}{4}$$

$$\frac{\pi}{4} (\pi - \ln 4)$$

Worked example

A finite region is bounded by the curve with equation $y = \frac{3}{10(2+5x)}$, the x -axis, and the lines $x = 1$ and $x = -2$.

Find the exact volume of the solid formed when the region is rotated through 360° about the x -axis.

Your turn

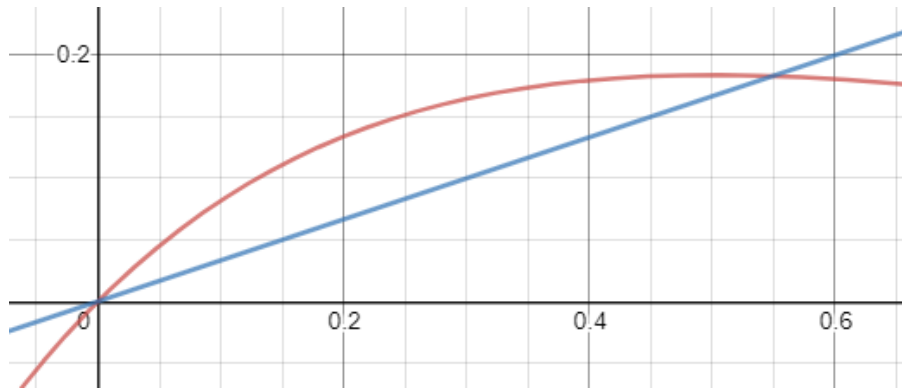
A finite region is bounded by the curve with equation $y = \frac{10}{3(5+2x)}$, the x -axis, and the lines $x = -1$ and $x = 2$.

Find the exact volume of the solid formed when the region is rotated through 360° about the x -axis.

$$\frac{100\pi}{81}$$

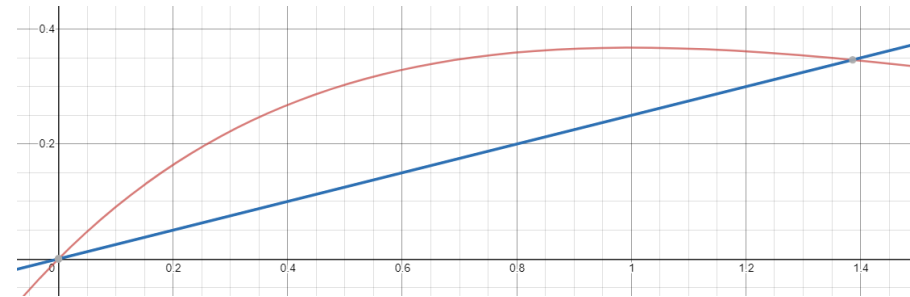
Worked example

A finite region is bounded by the curve with equation $y = xe^{-2x}$ and the line $y = \frac{1}{3}x$. Find the volume of the solid formed when the region is rotated through 360° about the x -axis.



Your turn

A finite region is bounded by the curve with equation $y = xe^{-x}$ and the line $y = \frac{1}{4}x$. Find the volume of the solid formed when the region is rotated through 360° about the x -axis.



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