

Core Pure 1

Roots of Polynomials

Course Overview

1. Use relationships between coefficients and roots of a quadratic, cubic or quartic equation.
2. Find the value of expressions based on the roots of a polynomial.
3. Find the new polynomial when the roots undergo a linear transformation.

2.1	Solve any quadratic equation with real coefficients. Solve cubic or quartic equations with real coefficients.	Given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics, for example: (i) $f(z) = 2z^3 - 5z^2 + 7z + 10$ Given that $2z - 3$ is a factor of $f(z)$, use algebra to solve $f(z) = 0$ completely. (ii) $g(x) = x^4 - x^3 + 6x^2 + 14x - 20$ Given $g(1) = 0$ and $g(-2) = 0$, use algebra to solve $g(x) = 0$ completely.
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Roots of Polynomials

The purpose of this chapter is to understand the underlying relationship between the **roots** of a polynomial, and the **coefficients** of each term.

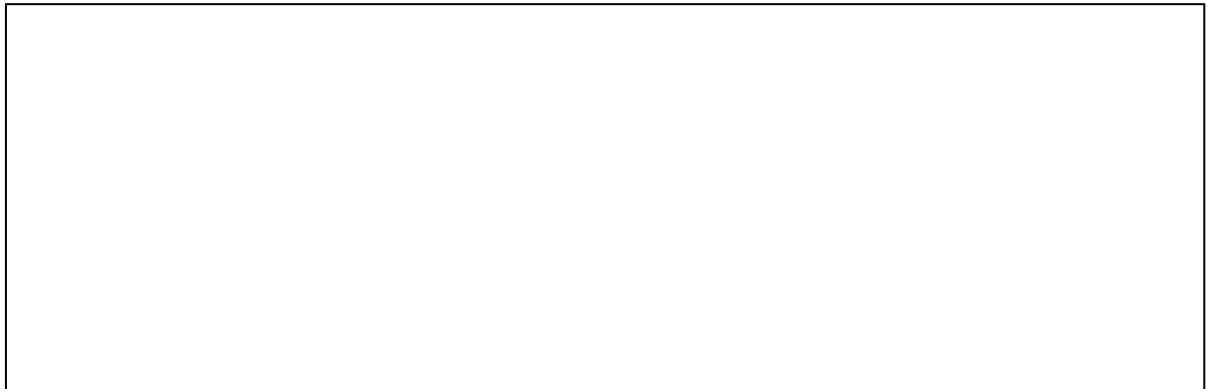
Roots of Quadratics

If α and β are the roots of a quadratic $ax^2 + bx + c$ then

$$ax^2 + bx + c \equiv a(x - \alpha)(x - \beta)$$

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This pattern generalises to higher order polynomials which will be discussed further later on.

Quadratic Example

The roots of the quadratic equation $2x^2 - 5x - 4 = 0$ are α and β . Without solving the equation, find the values of:

- (a) $\alpha + \beta$
- (b) $\alpha\beta$
- (c) $\frac{1}{\alpha} + \frac{1}{\beta}$
- (d) $\alpha^2 + \beta^2$

Question

The roots of the quadratic equation $3x^2 - 4x + 2 = 0$ are α and β . Without solving the equation, find the values of:

- (a) $\alpha + \beta$
- (b) $\alpha\beta$
- (c) $\frac{1}{\alpha} + \frac{1}{\beta}$
- (d) $\alpha^2 + \beta^2$

Example

The roots of a quadratic equation $ax^2 + bx + c = 0$ are $\alpha = -\frac{3}{2}$ and $\beta = \frac{5}{4}$.
Find integer values for a , b and c .

Test Your Understanding

1. For the quadratic $x^2 + 2x + 3$, find:

(a) The sum of the roots.

(b) The product of the roots.

2. If the roots of a quadratic equation $ax^2 + bx + c = 0$ are $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{5}$, determine integer values for a, b, c .