4A Roots of Quadratics

1. The roots of the quadratic equation $2x^2 - 5x - 4 = 0$ are α and β. Without solving the equation, find the values of:

a) $\alpha + \beta$

b) $\alpha\beta$

c) $\frac{1}{\alpha} + \frac{1}{\beta}$

d) $\alpha^2 + \beta^2$

2. The roots of the quadratic equation $ax^2 + bx + c = 0$ are $\alpha = -\frac{3}{2}$ and $\beta = \frac{5}{4}$. Find integer values for a, b and c.

4B Roots of Cubics

1. If α, β and γ are the roots of the equation $2x^3 + 3x^2 - 4x + 2 = 0$, find the values of:

a) $\alpha + \beta + \gamma$

b) $\alpha\beta + \beta\gamma + \gamma\alpha$

c) $\alpha\beta\gamma$

d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

2. The roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ are

$$\alpha = 1 - 2i, \beta = 1 + 2i \text{ and } \gamma = 2.$$

Find integer values for a, b, c and d.

4C Roots of Quartics

Patterns spotted for polynomials in general:

	Quadratics	Cubics	Quartics
Sum of 'singles'			
Sum of 'doubles'			
Sum of 'triples'			
Sum of 'quadruples'			

1. The equation $x^4 + 2x^3 + px^2 + qx - 60 = 0, x \in \mathbb{C}, \ \ p, q \in \mathbb{R}$, has roots α, β, γ and δ. Given that $\gamma = -2 + 4i$ and $\delta = \gamma^*$:

a) Show that $\alpha + \beta - 2 = 0$ and that $\alpha\beta + 3 = 0$

b) Hence, find all the roots of the equation and the values of p and q.

4D Expressions Relating to Roots of Polynomials

1.

a) Expand $(\alpha + \beta + \gamma)^2$

b) A cubic equation has roots α, β and γ such that $\alpha\beta + \beta\gamma + \gamma\alpha = 7$ and $\alpha + \beta + \gamma = -3$. Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

The sum of the squared singles	=	The square of the sum of the singles	- 2 x	The sum of the doubles

The sum of the cubed singles	=	The cube of the sum of the singles	- 3 x	The sum of the doubles	+ 3 x	The sum of the triples

2. The three roots of a cubic equation are α, β and γ.

   Given that $\alpha\beta\gamma = 4$, $\alpha\beta + \beta\gamma + \gamma\alpha = -5$ and $\alpha + \beta + \gamma = 3$,

   find the value of $(\alpha + 3)(\beta + 3)(\gamma + 3)$.

4E Linear Transformations of Roots

1. The cubic equation

$$x^3 - 2x^2 + 3x - 4 = 0$$

has roots α, β and γ. Find the equations of the polynomials with roots:

a) 2α, 2β and 2γ

Alternative approach by considering graphical transformations & substitution (easier)

b) $(\alpha + 3)$, $(\beta + 3)$ and $(\gamma + 3)$

2. The quartic equation $x^4 - 3x^3 + 15x + 1 = 0$ has roots α, β, γ and δ.
   Find the equation with roots $(2\alpha + 1)$, $(2\beta + 1)$, $(2\gamma + 1)$ and $(2\delta + 1)$.