**4A Roots of Quadratics**

1. The roots of the quadratic equation $2x^{2}-5x-4=0$ are $α$ and $β$. Without solving the equation, find the values of:
2. $α+β$
3. $αβ$
4. $\frac{1}{α}+\frac{1}{β}$
5. $α^{2}+β^{2}$
6. The roots of the quadratic equation $ax^{2}+bx+c=0$ are $α=-\frac{3}{2}$ and $β=\frac{5}{4}$. Find integer values for $a$, $b$ and $c$.

**4B Roots of Cubics**

1. If $α$, $β$ and $γ$ are the roots of the equation $2x^{3}+3x^{2}-4x+2=0$, find the values of:
2. $α+β+γ$
3. $αβ+βγ+γα$
4. $αβγ$
5. $\frac{1}{α}+\frac{1}{β}+\frac{1}{γ}$
6. The roots of a cubic equation $ax^{3}+bx^{2}+cx+d=0$ are

$α=1-2i$, $β=1+2i$ and $γ=2$.

Find integer values for $a$, $b$, $c$ and $d$.

**4C Roots of Quartics**

Patterns spotted for polynomials in general:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Quadratics | Cubics | Quartics |
| Sum of ‘singles’ |  |  |  |
| Sum of ‘doubles’ |  |  |  |
| Sum of ‘triples’ |  |  |  |
| Sum of ‘quadruples’ |  |  |  |

1. The equation $x^{4}+2x^{3}+px^{2}+qx-60=0$, $x\in C$, $p,q\in R$,

has roots $α$, $β$, $γ$ and $δ$. Given that $γ=-2+4i$ and $δ=γ^{\*}$:

1. Show that $α+β-2=0$ and that $αβ+3=0$
2. Hence, find all the roots of the equation and the values of $p$ and $q$.

**4D Expressions Relating to Roots of Polynomials**

1. Expand $\left(α+β+γ\right)^{2}$
2. A cubic equation has roots $α$, $β$ and $γ$ such that $αβ+βγ+γα=7$ and $α+β+γ=-3$. Find the value of $α^{2}+β^{2}+γ^{2}$.

The sum of the doubles

- 2 x

The square of the sum of the singles

=

The sum of the squared singles

The sum of the triples

+ 3 x

The sum of the doubles

- 3 x

The cube of the sum of the singles

=

The sum of the cubed singles

1. The three roots of a cubic equation are $α$, $β$ and $γ$.

Given that $αβγ=4$, $αβ+βγ+γα=-5$ and $α+β+γ=3$,

find the value of $\left(α+3\right)\left(β+3\right)\left(γ+3\right)$.

**4E Linear Transformations of Roots**

1. The cubic equation

$$x^{3}-2x^{2}+3x-4=0$$

has roots $α$, $β$ and $γ$. Find the equations of the polynomials with roots:

1. $2α$, $2β$ and $2γ$

Alternative approach by considering graphical transformations & substitution (easier)

1. $\left(α+3\right)$, $\left(β+3\right)$ and $\left(γ+3\right)$
2. The quartic equation $x^{4}-3x^{3}+15x+1=0$ has roots $α$, $β$, $γ$ and $δ$.

Find the equation with roots $\left(2α+1\right)$, $\left(2β+1\right)$, $\left(2γ+1\right)$ and $\left(2δ+1\right)$.