3E Integrating with Partial Fractions

1. Prove that:

$$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} ln \left| \frac{a + x}{a - x} \right| + c$$

2. Show that:

$$\int \frac{1+x}{x^3+9x}\,dx = A\ln\left(\frac{x^2}{x^2+9}\right) + B\arctan\left(\frac{x}{3}\right) + c$$

where A and B are constants to be found.

3.

a) Express the following as partial fractions

$$\frac{x^4 + x}{x^4 + 5x^2 + 6}$$

b) Hence, find:

$$\int \frac{x^4 + x}{x^4 + 5x^2 + 6}\, dx$$