3.5) Integrating using partial fractions

Worked example

Find:

$$\int \frac{1+x}{x^3+16x}\,dx$$

Your turn

Find:

$$\int \frac{1+x}{x^3+9x}\,dx$$

$$\frac{1}{18}\ln\left(\frac{x^2}{x^2+9}\right)+\frac{1}{3}\arctan\left(\frac{x}{3}\right)+c$$

Worked example

Find:

$$\int \frac{3x - x^2}{(x^2 + 9)(x + 3)}\, dx$$

Your turn

Find:

$$\int \frac{x^2 - 3x}{(x^2 + 6)(x + 2)}\, dx$$

$$\ln|x + 2| - \frac{3}{\sqrt{2}} \arctan\left(\frac{x}{\sqrt{6}}\right) + c$$

Worked example

Find:

$$\int \frac{x^4 + x}{x^4 + 7x^2 + 12}\, dx$$

Your turn

Find:

$$\int \frac{x^4 + x}{x^4 + 5x^2 + 6}\, dx$$

$$x + \frac{1}{2}\ln\left|\frac{x^2+2}{x^2+3}\right| + 2\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) - 3\sqrt{3}\arctan\left(\frac{x}{\sqrt{3}}\right) + c$$

Worked example

Evaluate:

$$\int_0^1 \frac{2}{(x+2)(x^2+2)}$$

Your turn

Find:

$$\int_0^1 \frac{2}{(x+1)(x^2+1)}$$

$$\frac{1}{4}(\pi + 2\ln 2)$$

Worked example

Find:

$$\int \frac{x^4+1}{x(x^2+3)^2}\,dx$$

Your turn

Find:

$$\int \frac{x^4+1}{x(x^2+2)^2}\,dx$$

$$\frac{1}{4}\ln|x| + \frac{3}{8}\ln|x^2+2| + \frac{5}{4(x^2+2)} + c$$

Worked example

Find:

$$\int \frac{1}{3x^2 + 6x + 7}\, dx$$

Your turn

Find:

$$\int \frac{1}{2x^2 + 4x + 11}\, dx$$

$$\frac{1}{3\sqrt{2}} \arctan\left(\frac{\sqrt{2}(x+1)}{3}\right) + c$$