3.4) Integrating with inverse trigonometric functions

Worked example

Use an appropriate substitution to show that:

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + C$$

Your turn

Use an appropriate substitution to show that:

$$\int \frac{1}{1-x^2}\,dx = \arcsin x + C$$

Proof

Worked example

Use an appropriate substitution to show that:

$$\int \frac{1}{a^2 - x^2}\,dx = \arcsin\frac{x}{a} + C$$

where a is a positive constant and $|x| < a$

Your turn

Use an appropriate substitution to show that:

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C, a > 0, |x| < a$$

Proof

Worked example

Find:

$$\int \frac{3}{2+x^2}\,dx$$

Your turn

Find:

$$\int \frac{4}{5+x^2}\,dx$$

$$\frac{4}{\sqrt{5}}\arctan\left(\frac{x}{\sqrt{5}}\right)+c$$

Worked example

Find:

$$\int \frac{1}{49 + 16x^2}\, dx$$

Your turn

Find:

$$\int \frac{1}{25 + 9x^2}\, dx$$

$$\frac{1}{15}\arctan\left(\frac{3x}{5}\right) + c$$

Worked example

Find:

$$\int_{-1}^{2} \frac{1}{\sqrt{21-3x^2}}\,dx$$

Your turn

Find:

$$\int_{-\frac{\sqrt{3}}{4}}^{\frac{\sqrt{3}}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx$$

$$\frac{\pi}{6}$$

Worked example

Find:

$$\int \frac{x+9}{\sqrt{1-9x^2}}\,dx$$

Your turn

Find:

$$\int \frac{x+4}{\sqrt{1-4x^2}}\,dx$$

$$\frac{1}{4}\sqrt{1-4x^2}+2\arcsin 2x+c$$

Worked example

Find:

$$\int \frac{6x-5}{3+x^2}\,dx$$

Your turn

Find:

$$\int \frac{8x-3}{4+x^2}\,dx$$

$$4\ln(4+x^2) - \frac{3}{2}\arctan\left(\frac{x}{2}\right) + c$$

Worked example

Find:

$$\int \frac{3x-1}{\sqrt{5-4x^2}}\,dx$$

Your turn

Find:

$$\int \frac{4x-1}{\sqrt{6-5x^2}}\,dx$$

$$-\frac{4}{5}\sqrt{6-5x^2}-\frac{1}{\sqrt{5}}\arcsin\left(\sqrt{\frac{5}{6}}x\right)+c$$