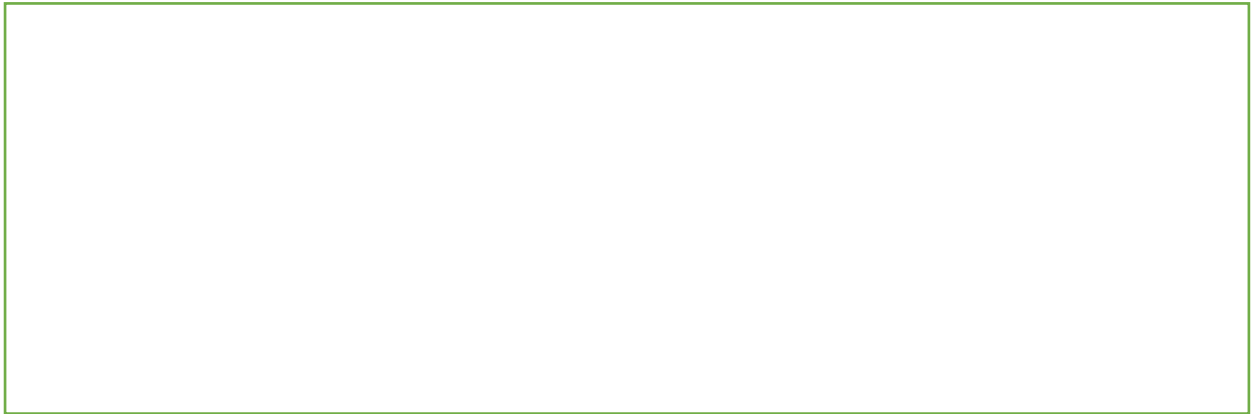


Sums of Squares and Cubes



Example

(a) Show that

$$\sum_{r=n+1}^{2n} r^2 = \frac{1}{6}n(2n+1)(7n+1)$$

(b) Verify that the result is true for $n = 1$ and $n = 2$.

Example:

Find the sum of the following series

$$\sum_{r=1}^n r(r+3)(2r-1)$$

and hence evaluate

$$\sum_{r=11}^{40} r(r+3)(2r-1)$$

Test Your Understanding

5. (a) Use the standard results for $\sum_{r=1}^n r$ and $\sum_{r=1}^n r^2$ to show that

$$\sum_{r=1}^n (r+2)(r+3) = \frac{1}{3}n(n^2 + 9n + 26)$$

for all positive integers n .

(6)

- (b) Hence show that

$$\sum_{r=n+1}^{3n} (r+2)(r+3) = \frac{2}{3}n(an^2 + bn + c)$$

where a , b and c are integers to be found.

(4)

2.

(a) Use the results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$, to prove that

$$\sum_{r=1}^n r(r+1)(r+5) = \frac{1}{4} n(n+1)(n+2)(n+7)$$

for all positive integers n .

(5)

(b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5).$$

(2)

Extension: Given that n is even, determine $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \dots - n^2$