Sums of Squares and Cubes

Example

(a) Show that

$$\sum_{r=n+1}^{2n} r^2 = \frac{1}{6}n(2n+1)(7n+1)$$

(b) Verify that the result is true for $n = 1$ and $n = 2$.

Example:

Find the sum of the following series

$$\sum_{r=1}^{n} r(r+3)(2r-1)$$

and hence evaluate

$$\sum_{r=11}^{40} r(r+3)(2r-1)$$

Test Your Understanding

5. (a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2+9n+26)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n}(r+2)(r+3) = \frac{2}{3}n(an^2+bn+c)$$

where a, b and c are integers to be found.

(4)

2.

(*a*) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4} n(n+1)(n+2)(n+7)$$

for all positive integers n. **(5)**

(*b*) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5).$$

(2)

Extension: Given that n is even, determine $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - \cdots - n^2$

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