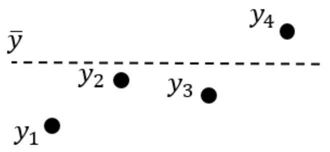
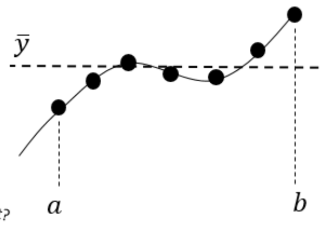
The Mean Value of a Function

How would we find the mean of a set of values y values $y_1, y_2, \dots, y_n$?



$$\bar{y} = \frac{y_1 + y_2 + \cdots + y_n}{n}$$

So the question then is, can we extend this to the continuous world, with a function $y = f(x)$, between $x = a$ and $x = b$?



continuous equivalent?

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$$\bar{y} = \frac{\int_a^b f(x)\,dx}{b - a}$$

Integration can be thought of as the continuous version of summation of the y values.

The width of the interval, $b - a$, could (sort of) be thought of as the number of points in the interval on an infinitesimally small scale.

✎ The **mean value** of the function $y = f(x)$ over the interval $[a, b]$ is given by

$$\frac{1}{b-a}\int_b^a f(x)dx$$

We write it as $\bar{y}$ or $\bar{f}$ or y_m.

Textbook Example

1. Find the mean value of $f(x) = \frac{4}{\sqrt{2+3x}}$ over the interval $[2,6]$.

2. $f(x) = \frac{4}{1+e^x}$

(a) Show that the mean value of $f(x)$ over the interval $[\ln 2, \ln 6]$ is $\frac{4\ln\frac{9}{7}}{\ln 3}$

(b) Use your answer to part a to find the mean value over the interval $[\ln 2, \ln 6]$ of $f(x) + 4$.

Use geometric considerations to write down the mean value of $-f(x)$ over the interval $[\ln 2, \ln 6]$