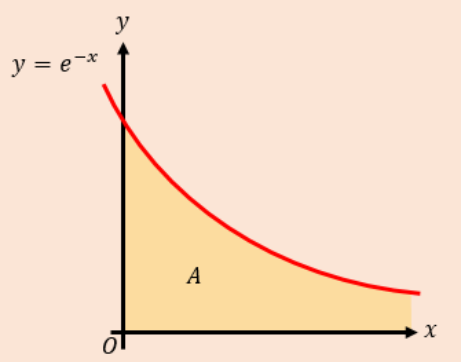
3A Improper Integrals

1. Calculate the area indicated in the diagram



2. Evaluate the integral below, or show that it is not convergent.

a)

$$\int_1^{\infty} \frac{1}{x^2}\ dx$$

b)

$$\int_1^{\infty} \frac{1}{x}\ dx$$

c)

$$\int_0^1 \frac{1}{x^2}\ dx$$

d)

$$\int_0^2 \frac{x}{\sqrt{4-x^2}}\ dx$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

3.

a) Find $\int xe^{-x^2}\,dx$

b) Hence, show that $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ converges, and find its value

A final thought on positive and negative areas and the difference between ‘find the integral’, and ‘find the area’