

3.1) Improper integrals

Worked example

Find the value of the improper integral

$$\int_1^{\infty} \frac{1}{x^3} dx$$

Your turn

Find the value of the improper integral

$$\int_1^{\infty} \frac{1}{x^2} dx$$

1

Worked example

Find the value of the improper integral

$$\int_2^{\infty} x^{-\frac{5}{2}} dx$$

Your turn

Find the value of the improper integral

$$\int_2^{\infty} x^{-\frac{3}{2}} dx$$

$$\sqrt{2}$$

Worked example

Find the value of the improper integral

$$\int_0^{\infty} e^{-2x} dx$$

Your turn

Find the value of the improper integral

$$\int_0^{\infty} e^{-3x} dx$$

$$\frac{1}{3}$$

Worked example

Show that the integral does not converge:

$$\int_0^1 \frac{1}{x^3} dx$$

Your turn

Show that the integral does not converge:

$$\int_0^1 \frac{1}{x^2} dx$$

Shown

Worked example

Show that the integral does not converge:

$$\int_1^{\infty} \frac{1}{\sqrt[3]{x}} dx$$

Your turn

Show that the integral does not converge:

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx$$

Shown

Worked example

Show that the integral converges and find its value:

$$\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$$

Your turn

Show that the integral converges and find its value:

$$\int_{-\infty}^{\infty} x e^{-x^2} dx$$

0

Worked example

Evaluate the integral:

$$\int_0^2 \frac{6x}{\sqrt[3]{4-x^2}} dx$$

Your turn

Evaluate the integral:

$$\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$$

2

Worked example

Show that the integral is divergent:

$$\int_0^{\frac{\pi}{2}} \tan x \, dx$$

Your turn

Show that the integral is divergent:

$$\int_0^{\pi} \sec^2 x \, dx$$

Shown

Worked example

Find the exact value of

$$\int_0^{\infty} \frac{1}{3x^2 + 4x + 1} dx$$

Your turn

Find the exact value of

$$\int_0^{\infty} \frac{1}{2x^2 + 3x + 1} dx$$

ln 2