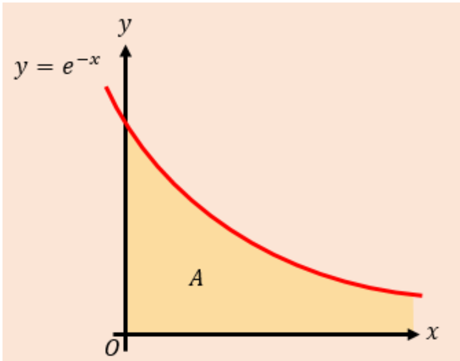
3A Improper Integrals

1. Calculate the area indicated in the diagram



2. Evaluate the integral below, or show that it is not convergent.

a)

$$\int_1^\infty \frac{1}{x^2}\,dx$$

b)

$$\int_1^\infty \frac{1}{x}\,dx$$

c)

$$\int_0^1 \frac{1}{x^2}\ dx$$

d)

$$\int_0^2 \frac{x}{\sqrt{4-x^2}}\ dx$$

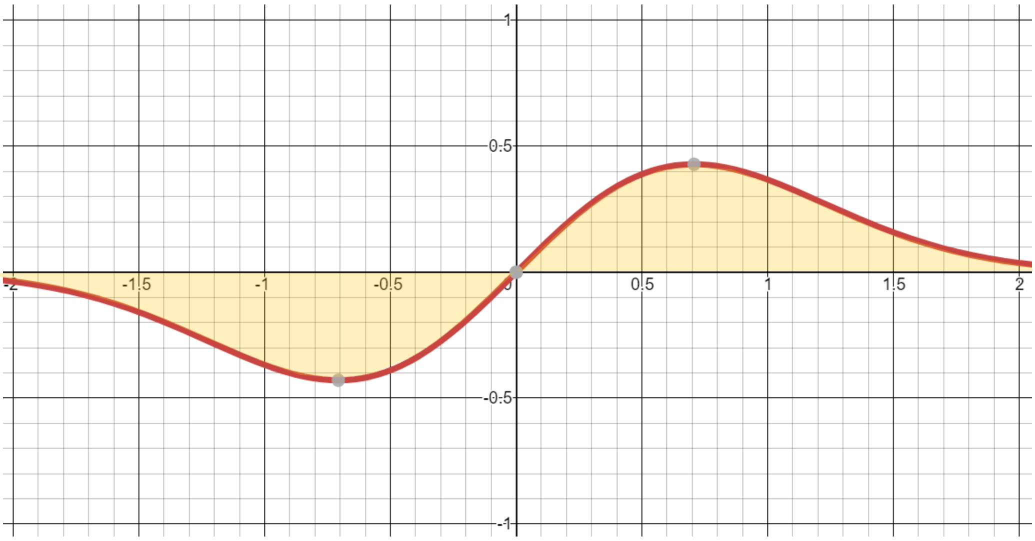
$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

3.

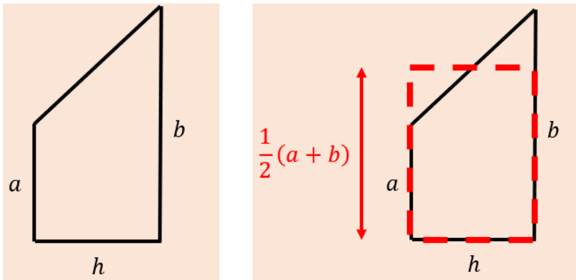
a) Find $\int xe^{-x^2}\ dx$

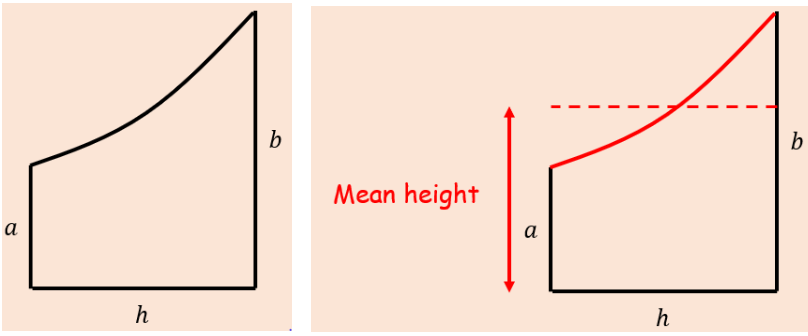
b) Hence, show that $\int_{-\infty}^{\infty} xe^{-x^2}\ dx$ converges, and find its value

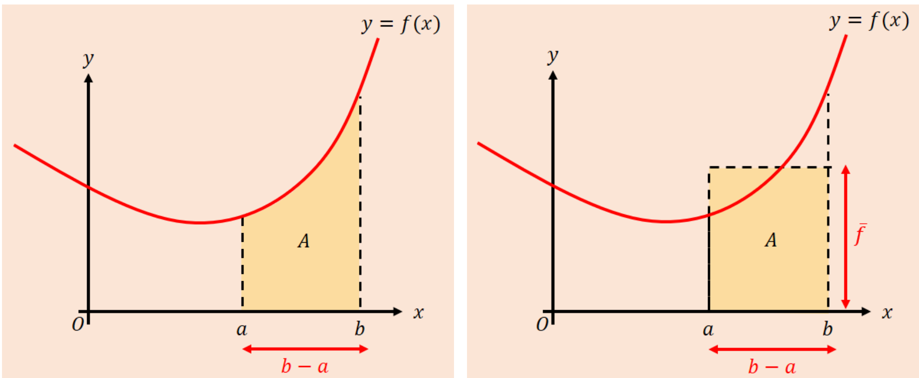
A final thought on positive and negative areas and the difference between ‘find the integral’, and ‘find the area’



3B Mean Value of a Function







1. Find the mean value of $f(x) = \frac{4}{\sqrt{2+3x}}$ in the interval $[2,6]$.

2. Given that $f(x) = \frac{4}{1+e^x}$

a) Show that the mean value of $f(x)$ on the interval $[ln2,\ ln6]$ is

$$\frac{4ln\frac{9}{7}}{ln3}$$

b) Use your answer to part a) to find the mean value of $f(x) + 4$ over the interval $[ln2, ln6]$

c) Use geometric considerations to write down the mean value of $y = -f(x)$ over the interval $[ln2, ln6]$

In General:

Vertical Transformations:

Horizontal Transformations:

3C Differentiating Inverse Trig Functions

1. Show that $\frac{d}{dx}(arcsinx) = \frac{1}{\sqrt{1-x^2}}$

2. $\frac{d}{dx}(arccosx) = -\frac{1}{\sqrt{1-x^2}}$

3. Find $\frac{d}{dx}(arctanx)$

4. Given $y = arcsinx^2$, find $\frac{dy}{dx}$

a) Using implicit differentiation

b) Using the chain rule and the formula for $\frac{d}{dx}(arcsinx)$

A Key point on the chain rule above: don’t just multiply by the derivative! (as x does not remain x in the derivative)

5. Given $y = arctan\left(\frac{1-x}{1+x}\right)$, find $\frac{dy}{dx}$

6. Show that

$$sin(arccosx) = \sqrt{1 - x^2}$$

3D Integrating with Trig Substitutions

1.

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx$$

2. Find the integral:

$$\int \frac{1}{a^2 + x^2}\, dx$$

A reminder of the formula book

$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\arcsin\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)$

3. Find

$$\int \frac{4}{5 + x^2}\, dx$$

4. Find

$$\int \frac{1}{25 + 9x^2}\, dx$$

5. Evaluate the following, leaving your answer in terms of π.

$$\int_{-\frac{\sqrt{3}}{4}}^{\frac{\sqrt{3}}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx$$

6. Find

$$\int \frac{x + 4}{\sqrt{1 - 4x^2}}\, dx$$

3E Integrating with Partial Fractions

1. Prove that:

$$\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} ln \left|\frac{a + x}{a - x}\right| + c$$

2. Show that:

$$\int \frac{1+x}{x^3+9x}\,dx = A\ln\left(\frac{x^2}{x^2+9}\right) + B\arctan\left(\frac{x}{3}\right) + c$$

where A and B are constants to be found.

3.

a) Express the following as partial fractions

$$\frac{x^4 + x}{x^4 + 5x^2 + 6}$$

b) Hence, find:

$$\int \frac{x^4 + x}{x^4 + 5x^2 + 6}\, dx$$