3) Series

3.1) Sums of natural numbers

Chapter CONTENTS

Worked example

Evaluate these summations by writing out the elements:

$$\sum_{r=1}^{10}(3r-2)$$

$$\sum_{r=1}^{5}(2-3r)$$

Your turn

Evaluate these summations by writing out the elements:

$$\sum_{r=1}^{10}(2r-3)$$

$-1+1+3+5+7+9+11+13+15+17=80$

Worked example

Evaluate these summations by writing out the elements:

$$\sum_{r=1}^{6} r^4$$

$$\sum_{r=2}^{7} (r^3)$$

Your turn

Evaluate these summations by writing out the elements:

$$\sum_{r=3}^{8} r^2$$

$3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 = 199$

Worked example

Evaluate:

$$\sum_{r=1}^{100} r$$

$$\sum_{r=20}^{50} r$$

Your turn

Evaluate:

$$\sum_{r=21}^{50} r$$

1065

Worked example

Show that

$$\sum_{r=5}^{3N-1} r = \frac{9}{2}N^2 - \frac{3}{2}N - 10$$

(for $N \geq 2$)

Your turn

Show that

$$\sum_{r=5}^{2N-1} r = 2N^2 - N - 10$$

(for $N \geq 3$)

Shown

Worked example

Evaluate:

$$\sum_{r=1}^{100}(3r-2)$$

$$\sum_{r=1}^{50}(2-3r)$$

Your turn

Evaluate

$$\sum_{r=1}^{100}(2r-3)$$

9800

Worked example

Show that:

$$\sum_{r=1}^{n}(5r-3)=\frac{1}{2}n(2n+5)$$

Hence evaluate

$$\sum_{r=50}^{100}(5r-3)$$

Your turn

Show that:

$$\sum_{r=1}^{n}(7r-4)=\frac{1}{2}n(7n-1)$$

Hence evaluate

$$\sum_{r=20}^{50}(7r-4)$$

Shown

7471

Worked example

Find the smallest value of k for which

$$\sum_{r=1}^{k}(6r-2) > 310$$

Your turn

Find the smallest value of k for which

$$\sum_{r=1}^{k}(4r-5) > 4850$$

$k = 51$

Worked example

Given that

$$\sum_{r=1}^{n} f(r) = 3n^2 + 4n$$

deduce an expression for $f(r)$ in terms of r

Your turn

Given that

$$\sum_{r=1}^{n} f(r) = 2n^2 + 5n$$

deduce an expression for $f(r)$ in terms of r

$$f(r) = 4r + 3$$

Worked example

$f(r) = ar + b$, where a and b are rational constants.

Given that

$$\sum_{r=1}^{8} f(r) = 152$$

and

$$\sum_{r=1}^{12} f(r) = 324$$

find an expression for $f(r)$

Your turn

$f(r) = ar + b$, where a and b are rational constants.

Given that

$$\sum_{r=1}^{4} f(r) = 36$$

and

$$\sum_{r=1}^{6} f(r) = 78$$

find an expression for $f(r)$

$$f(r) = 4r - 1$$

3.2) Sums of squares and cubes

Chapter CONTENTS

Worked example

Evaluate:

$$\sum_{r=1}^{100} r^2$$

$$\sum_{r=20}^{50} r^2$$

Your turn

Evaluate:

$$\sum_{r=21}^{100} r^2$$

335480

Worked example

Evaluate:

$$\sum_{r=1}^{100} r^3$$

$$\sum_{r=20}^{50} r^3$$

Your turn

Evaluate:

$$\sum_{r=21}^{100} r^3$$

25458400

Worked example

Show that

$$\sum_{r=2n+1}^{4n} r^2 = \frac{1}{3}n(4n+1)(14n+1)$$

Your turn

Show that

$$\sum_{r=n+1}^{2n} r^2 = \frac{1}{6}n(2n+1)(7n+1)$$

Shown

Worked example

Show that

$$\sum_{r=1}^{n}(r+2)(r+3)=\frac{1}{3}n(n^2+9n+26)$$

Your turn

Show that

$$\sum_{r=1}^{n}(r+2)(r-1)=\frac{1}{3}n(n+4)(n-1)$$

Shown

Worked example

Show that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

and hence evaluate

$$\sum_{r=10}^{40} r(r+1)(r+5)$$

Your turn

Show that

$$\sum_{r=1}^{n} r(r+3)(2r-1) = \frac{1}{6}n(n+1)(3n^2+13n-4)$$

and hence evaluate

$$\sum_{r=11}^{40} r(r+3)(2r-1)$$

Shown

1445230

Worked example

Find the value of n that satisfies

$$\sum_{r=1}^{n}(r^2 - r) = \sum_{r=1}^{n} 5$$

Your turn

Find the value of n that satisfies

$$\sum_{r=1}^{n}(r^3 - r^2) = \sum_{r=1}^{n} 7r$$

$n = 4$