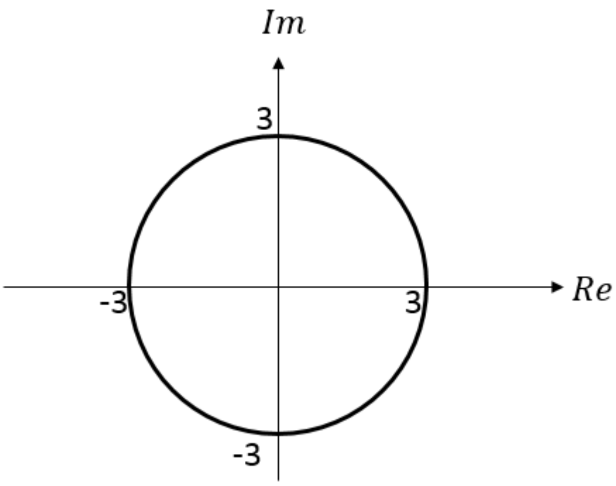
Loci

You have already encountered loci at GCSE as **a set of points (possibly forming a line or region) which satisfy some restriction**.

The definition of a circle for example is "a set of points equidistant from a fixed centre".

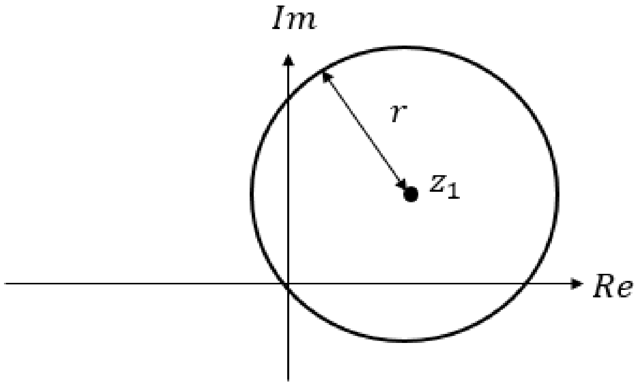


$|z| = 3$ means that the modulus of the complex number has to be 3. What points does this give us on the Argand diagram?

A quick reminder...

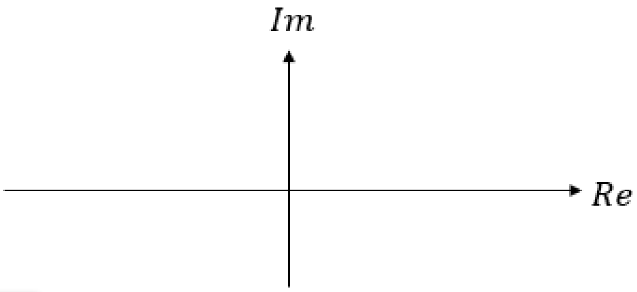
$$|z| = |\boldsymbol{x} + \boldsymbol{iy}|$$
$$= \boxed{}$$
$$|z - 3| = |\boldsymbol{x} + \boldsymbol{iy} - \boldsymbol{3}|$$
$$= \boxed{}$$

Loci of form $|z - z_1| = r$



What does $|z - z_1| = r$ mean?

Sketch the locus of points represented by $|z - 5 - 3i| = 3$



Find the Cartesian equation of this locus.

Questions:

Try Page 34 , Ex 2E Q1

Q2: Worked Example:

2 Given that z satisfies $|z - 5 - 4i| = 8$,

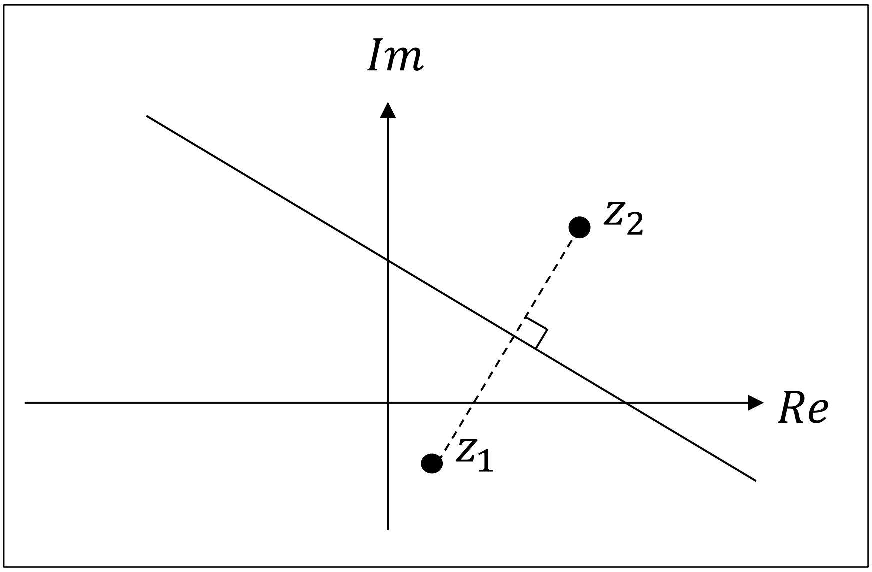
a sketch the locus of z on an Argand diagram

b find the exact values of z that satisfy:

i both $|z - 5 - 4i| = 8$ and $\text{Re}(z) = 0$ **ii** both $|z - 5 - 4i| = 8$ and $\text{Im}(z) = 0$

Loci of form $|z - z_1| = |z - z_2|$

What does $|z - z_1| = |z - z_2|$ mean?



Example,

Sketch the locus of points represented by $|z| = |z - 6i|$.
Write its equation.

Test Your Understanding So Far

Find the Cartesian equation of the locus of z if $|z-3| = |z+i|$, and sketch the locus of z on an Argand diagram.

What if we also required that $Re(z) = 0$?

Minimising/Maximising $\arg(z)$ and $|z|$

A complex number z is represented by the point P. Given that $|z - 5 - 3i| = 3$

(a) Sketch the locus of P

(b) Find the Cartesian equation of the locus.

(c) Find the maximum value of $\arg z$ in the interval $(-\pi, \pi)$

(d) Find the minimum and maximum values of $|z|$

We did this earlier...

a)

b)

c)

d)

Quickfire Test Your Understanding

Given that the complex number z satisfies the equation $|z - 12 - 5i| = 3$, find the minimum value of $|z|$ and the maximum.

Minimising $|z|$ with perpendicular bisectors

(From earlier) Find the Cartesian equation of the locus of z if $|z - 3| = |z + i|$, and sketch the locus of z on an Argand diagram.

Hence, find the least possible value of $|z|$.

$\arg(z - z_1) = \theta$

$\arg(z) = \frac{\pi}{6}$?

$\arg(z + 3 + 2i) = \frac{3\pi}{4}$?