# 2.4) Loci in the Argand diagram

## Worked example

Sketch the locus of points represented by $|z| = 3$

$|z| = 4$

## Your turn

Sketch the locus of points represented by $|z| = 5$

Circle centre $(0, 0)$ radius $5$

Cartesian equation: $x^2 + y^2 = 25$

## Worked example

Sketch the locus of points represented by

$$|z + 3 - 5i| = 2$$

and find its Cartesian equation

Sketch the locus of points represented by

$$|z - 3 + 5i| = 4$$

and find its Cartesian equation

## Your turn

Draw the locus of points that satisfy:

$$|z - 5 - 3i| = 6$$

and find its Cartesian equation

Circle centre $(5, 3)$ radius $6$

$$(x - 5)^2 + (y - 3)^2 = 36$$

# Worked example

Sketch the locus of points represented by

$$|3 - z| = 5$$

and find its Cartesian equation

Sketch the locus of points represented by

$$|2i - z| = 4$$

and find its Cartesian equation

Sketch the locus of points represented by

$$|2 - 3i - z| = 2$$

and find its Cartesian equation

# Your turn

Draw the locus of points that satisfy:

$$|3 - 2i - z| = 3$$

and find its Cartesian equation

Circle centre $(3, -2)$ radius 3

$$(x - 2)^2 + (y + 2)^2 = 9$$

## Worked example

A complex number $z$ is represented by the point $P$. Given that $|z - 3 + 5i| = 2$

(a) Sketch the locus of $P$

(b) Find the Cartesian equation of the locus.

(c) Find the maximum value of $\arg z$ in the interval $(-\pi, \pi)$

(d) Find the minimum and maximum values of $|z|$

## Your turn

A complex number $z$ is represented by the point $P$. Given that $|z - 5 - 3i| = 3$

(a) Sketch the locus of $P$

(b) Find the Cartesian equation of the locus.

(c) Find the maximum value of $\arg z$ in the interval $(-\pi, \pi)$

(d) Find the minimum and maximum values of $|z|$

(a) Circle centre $(5, 3)$ radius 3

(b) $(x - 5)^2 + (y - 3)^2 = 36$

(c) $1.08$ (3 sf)

(d) Max $|z| = \sqrt{34} + 3$

Min $|z| = \sqrt{34} - 3$

## Worked example

Sketch the locus of points represented by

$$|z| = |z + 4i|$$

and find its Cartesian equation

Sketch the locus of points represented by

$$|z| = |z - 5|$$

and find its Cartesian equation

## Your turn

Sketch the locus of points represented by

$$|z| = |z - 6i|$$

and find its Cartesian equation

Perpendicular bisector of $(0, 0)$ and $(0, 6)$

$$y = 3$$

## Worked example

Sketch the locus of points represented by

$$|z - 3i| = |z + 1|$$

and find its Cartesian equation

## Your turn

Sketch the locus of points represented by

$$|z - 3| = |z + i|$$

and find its Cartesian equation

Perpendicular bisector of $(3, 0)$ and $(0, -1)$

$$y = -3x + 4$$

## Worked example

Find the Cartesian equation of the locus of $z$ if $|z - 3i| = |z + 1|$ and sketch the locus of $z$ on an Argand diagram.
Hence, find the least possible value of $|z|$.

## Your turn

Find the Cartesian equation of the locus of $z$ if $|z - 3| = |z + i|$, and sketch the locus of $z$ on an Argand diagram.
Hence, find the least possible value of $|z|$.

$$\frac{2\sqrt{10}}{5}$$

## Worked example

Given that the complex number $z$ satisfies the equation $|z - 8 + 6i| = 5$, find the minimum value of $|z|$ and the maximum.

## Your turn

Given that the complex number $z$ satisfies the equation $|z - 12 - 5i| = 3$, find the minimum value of $|z|$ and the maximum.

Minimum $= 10$
Maximum $= 16$

## Worked example

Sketch the locus of points represented by

$$\arg(z) = \frac{\pi}{4}$$

and find its Cartesian equation

Sketch the locus of points represented by

$$\arg(z) = \frac{\pi}{3}$$

and find its Cartesian equation

## Your turn

Sketch the locus of points represented by

$$\arg(z) = \frac{\pi}{6}$$

and find its Cartesian equation

Half-line from origin $(0, 0)$

$$y = \frac{1}{\sqrt{3}}x, \qquad x > 0, y > 0$$

## Worked example

Sketch the locus of points represented by

$$\arg(z - 2 + 3i) = \frac{\pi}{4}$$

and find its Cartesian equation

## Your turn

Sketch the locus of points represented by

$$\arg(z + 3 + 2i) = \frac{3\pi}{4}$$

and find its Cartesian equation

Half-line from $(-3, -2)$

$y = -x - 5, \qquad x < -3, y > -2$

## Worked example

Find the complex number $z$ which satisfies both $|z + 3 - 2i| = 50$ and $\arg(z + 3 - 2i) = \frac{\pi}{4}$

## Your turn

Find the complex number $z$ which satisfies both $|z + 3 + 2i| = 10$ and $\arg(z + 3 + 2i) = \frac{3\pi}{4}$

$z = \left(-3 - 5\sqrt{2}\right) + i(-2 + 5\sqrt{2})$

## Worked example

If the complex number $z$ satisfies both $\arg z = \frac{\pi}{4}$ and $\arg(z-3) = \frac{\pi}{2}$,

(a) Find the value of $z$

(b) Hence, find $\arg(z-6)$

## Your turn

If the complex number $z$ satisfies both $\arg z = \frac{\pi}{3}$ and $\arg(z-4) = \frac{\pi}{2}$,

(a) Find the value of $z$

(b) Hence, find $\arg(z-8)$

(a) $z = 4 + 4\sqrt{3}i$

(b) $\frac{2\pi}{3}$

| Worked example | Your turn |
| --- | --- |
| Given $\lvert z+4-8i\rvert=3$, show that the maximum value of $\arg(z+12-5i)$ in the interval $(-\pi,\pi)$ is $2\arcsin\left(\frac{3}{\sqrt{73}}\right)$ | Given $\lvert z+8-4i\rvert=2$, show that the maximum value of $\arg(z+15-2i)$ in the interval $(-\pi,\pi)$ is $2\arcsin\left(\frac{2}{\sqrt{53}}\right)$<br><br>Shown |