Multiplying and Dividing Complex Numbers

Find the modulus and argument of $1 + i$ and $1 + \sqrt{3}$.
When you multiply these complex numbers together, do you notice anything about the modulus and argument of the result?

	$\mathbf{1+i}$	$\mathbf{1+i\sqrt{3}}$	$\mathbf{(1+i)(1+i\sqrt{3})}$ $=$
r			
θ			

Observation: The moduli have been multiplied, and the arguments have been added!

Proof (not assessed):

Recall from Pure Year 2 that:

$$sin(a+b) = sin\ a\ cos\ b + cos\ a\ sin\ bcos(a+b)$$
$$= cos\ a\ cos\ b - sin\ a\ sin\ b$$

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_1 = r_1(cos\ \theta_2 + i\ sin\ \theta_2)$.

Then $z_1 z_2 = r_1 r_2[(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\sin\theta_1\ cos\ \theta_2 + sin\ \theta_1\cos\theta_2)]$

$$= r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$$

So $|z_1 z_2| = r_1 r_2 = |z_1||z_2|$

and $arg(z_1 z_2) = \theta_1 + \theta_2 = arg(z_1) + arg(z_2)$

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Find the modulus and argument of $z_1 z_2$ where $z_1 = 2 + 3i$ and $z_2 = 1 + \sqrt{2}\, i$

Find the product of $z_1 = 5(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$ and $z_2 = 4(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$

Express the following in the form x + iy

$z_1 = 4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

$z_2 = 7(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3})$

Remember: cos(-x) = cos(x) and sin(-x) = -sin(x)

[Textbook] Express $\frac{\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)}{2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)}$ in the form $x+iy$