

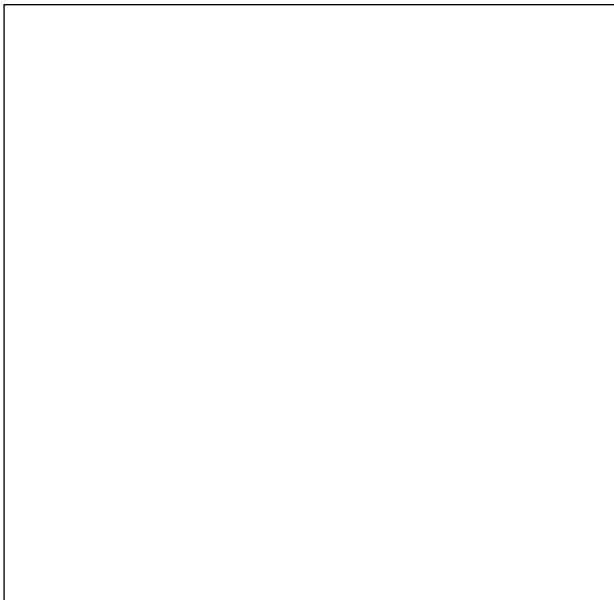
Multiplying and Dividing Complex Numbers

Find the modulus and argument of $1 + i$ and $1 + i\sqrt{3}$.

When you multiply these complex numbers together, do you notice anything about the modulus and argument of the result?

	$1 + i$	$1 + i\sqrt{3}$	$(1 + i)(1 + i\sqrt{3})$ = <input type="text"/>
r	<input type="text"/>	<input type="text"/>	<input type="text"/>
θ	<input type="text"/>	<input type="text"/>	<input type="text"/>

Observation: The moduli have been multiplied, and the arguments have been added!



Proof (not assessed):

Recall from Pure Year 2 that:

$$\begin{aligned} \sin(a + b) &= \sin a \cos b + \cos a \sin b \\ \cos(a + b) &= \cos a \cos b - \sin a \sin b \end{aligned}$$

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$.

$$\text{Then } z_1 z_2 = r_1 r_2 [(\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2) + i(\sin \theta_1 \cos \theta_2 + \sin \theta_1 \cos \theta_2)]$$

$$= r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

$$\text{So } |z_1 z_2| = r_1 r_2 = |z_1| |z_2|$$

$$\text{and } \arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$$

Multiplying and Dividing Complex Numbers

Find the modulus and argument of $z_1 z_2$ where $z_1 = 2 + 3i$ and $z_2 = 1 + \sqrt{2}i$

Find the product of $z_1 = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ and $z_2 = 4\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Express the following in the form $x + iy$

$$z_1 = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$z_2 = 7\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$$

Remember: $\cos(-x) = \cos(x)$ and $\sin(-x) = -\sin(x)$

[Textbook] Express $\frac{\sqrt{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)}{2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)}$ in the form $x + iy$