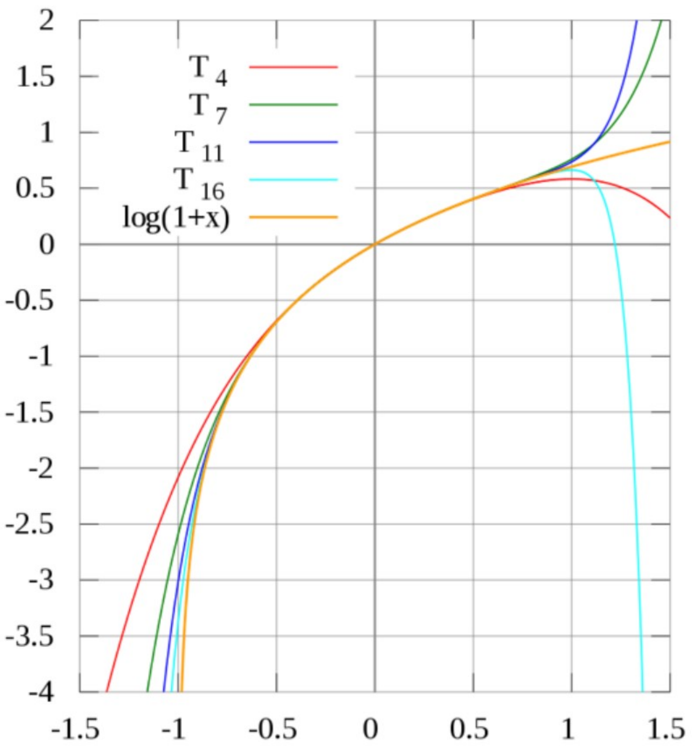

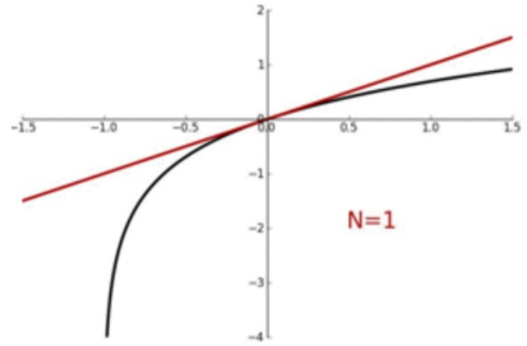

We previously said that we can only guarantee the curve is the same, i.e. the expansion is valid, 'around' $x = 0$.
For e^x and $\cos x$ we got lucky in that the curve turned out to be the same everywhere, for all x.

But as per the animation above, we can see that away from $x = 0$, the curve actually gets worse with more terms in the expansion!
Looking at the graph on the left, for what range of x is this expansion valid for?

$$\boldsymbol{-1 \leq x < 1}$$

Ex 2C

Composite Functions

✏ Standard Expansions (given in formula booklet)

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^r}{r!} + \cdots$ (valid for all x)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r-1}\frac{x^r}{r} + \cdots$ $\quad -1 < x \leq 1$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \cdots$ (valid for all x)

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^r x^{2r}}{2r!} + \cdots$ (valid for all x)

$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + \frac{(-1)^r x^{2r+1}}{2r+1} - \cdots$ $\quad -1 \leq x \leq 1$

We can also apply these when the input to the function is different.

$$\cos(2x^2) =$$

You *might* need some manipulation first.

Find the first three non-zero terms of the series expansion of $\ln\left(\frac{\sqrt{1+2x}}{1-3x}\right)$, and state the interval in x for which the expansion is valid.

Standard Expansions (given in formula booklet)

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{r-1}\frac{x^r}{r} + \cdots \qquad -1 < x \leq 1$

Given that terms in $x^n, n > 4$ may be neglected, use the series for e^x and $\sin x$ to show that $e^{\sin x} \approx 1 + x + \frac{x^2}{2}$

Standard Expansions (given in formula booklet)

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^r}{r!} + \cdots$ (valid for all x)

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \cdots$ (valid for all x)

Ex 2D