# 2.4) Series expansions of compound functions

## Worked example

Find the Maclaurin series for $e^{4x}$ up to and including the term in $x^4$

## Your turn

Find the Maclaurin series for $e^{3x}$ up to and including the term in $x^4$

$$e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2} + \frac{27x^4}{8} + \cdots$$

## Worked example

Find the Maclaurin series for $\ln(1+3x)$ up to and including the term in $x^4$

## Your turn

Find the Maclaurin series for $\ln(1+2x)$up to and including the term in $x^4$

$$\ln(1+2x) = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4 + \cdots$$

## Worked example

Write down the first four non-zero terms in the series expansion, in ascending powers of $x$, of

$$\sin(4x^3)$$

## Your turn

Write down the first four non-zero terms in the series expansion, in ascending powers of $x$, of

$$\cos(2x^2)$$

$$1 - 3x^4 + \frac{2}{3}x^8 - \frac{4}{45}x^{12} + \cdots$$

## Worked example

Find the first three non-zero terms of the series expansion of $\ln\left(\frac{\sqrt{1+3x}}{1-2x}\right)$, and state the interval in $x$ for which the expansion is valid.

## Your turn

Find the first three non-zero terms of the series expansion of $\ln\left(\frac{\sqrt{1+2x}}{1-3x}\right)$, and state the interval in $x$ for which the expansion is valid.

$$4x + \frac{7}{2}x^2 + \frac{31}{3}x^3 + \cdots$$

Valid for $-\frac{1}{3} \leq x < \frac{1}{3}$

## Worked example

Find the first three terms in the Maclaurin series expansion of $e^{\cos x}$

## Your turn

Find the first three terms in the Maclaurin series expansion of $e^{\sin x}$

$$1 + x + \frac{x^2}{2}$$

## Worked example

Find the series expansions, up to and including the term in $x^4$, of:

$$\ln(1 + 2x - 3x^2)$$

State the range of values of $x$ for which the expansion is valid

## Your turn

Find the series expansions, up to and including the term in $x^4$, of:

$$\ln(1 + x - 2x^2)$$

State the range of values of $x$ for which the expansion is valid

$$x - \frac{5x^2}{2} + \frac{7x^3}{3} - \frac{17x^4}{4} + \cdots$$

$$-\frac{1}{2} < x \leq \frac{1}{2}$$

## Worked example

Find the series expansions, up to and including the term in $x^4$, of:

$$\ln(16 + 8x + x^2)$$

State the range of values of $x$ for which the expansion is valid

## Your turn

Find the series expansions, up to and including the term in $x^4$, of:

$$\ln(9 + 6x + x^2)$$

State the range of values of $x$ for which the expansion is valid

$$2\ln 3 + \frac{2x}{3} - \frac{x^2}{9} + \frac{2x^3}{81} - \frac{x^4}{162} + \cdots$$

$$-3 < x \leq 3$$

## Worked example

Find the series expansion, up to and including the term in $x^4$, of:

$$\ln(16 + 8x + x^2)$$

State the range of values of $x$ for which the expansion is valid

## Your turn

Find the series expansion, up to and including the term in $x^4$, of:

$$\ln(9 + 6x + x^2)$$

State the range of values of $x$ for which the expansion is valid

$$2\ln 3 + \frac{2x}{3} - \frac{x^2}{9} + \frac{2x^3}{81} - \frac{x^4}{162} + \cdots$$

$$-3 < x \leq 3$$

## Worked example

Using the first two terms, $x - \frac{x^3}{3}$, in the expansion of $\arctan x$, find the first four terms of $e^{\arctan x}$

Deduce the first four terms in the series expansion of $e^{-\arctan x}$

## Your turn

Using the first two terms, $x + \frac{x^3}{3}$, in the expansion of $\tan x$, find the first four terms of $e^{\tan x}$

Deduce the first four terms in the series expansion of $e^{-\tan x}$

$$e^{\tan x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{2} + \cdots$$

$$e^{-\tan x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{2} + \cdots$$