2C Maclaurin Series

1. Given that $f(x) = e^x$ can be written in the form:

$$e^x = a_0 + a_1x + a_2x^2 + a_3x^3 + \ ...\ + a_rx^r$$

And that it is valid to differentiate an infinite series term by term, show that:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ ...\ + \frac{x^r}{r!}$$

Generalising:

2.

a) Express $\ln(1 + x)$ as an infinite series in ascending powers of x, up to and including the term in x^3

b) Using this series, find approximate values for:

i) $\ln(1.05)$

ii) $\ln(1.25)$

iii) $\ln(1.8)$

3. Find the Maclaurin expansion for $\sin x$, up to the term in x^5. Then use your expansion to find an approximation for $\sin 10^{\circ}$.

4. Find the Maclaurin expansion for $\cos x$, up to the term in x^4.

5. Proving Euler's relation:

$$e^{i\theta} = cos\theta + isin\theta$$