2.2) Higher derivatives

Worked example

Given that $y = \ln(1 + x)$, find the value of $\frac{d^3y}{dx^3}$ when $x = \frac{1}{2}$

Your turn

Given that $y = \ln(1 - x)$, find the value of $\frac{d^3y}{dx^3}$ when $x = \frac{1}{2}$

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Worked example

Given that $y = \sec 3x$, find the value of $\frac{d^3y}{dx^3}$ when $x = \frac{\pi}{4}$

Your turn

Given that $y = \sin^2 3x$, find the value of $\frac{d^4y}{dx^4}$ when $x = \frac{\pi}{6}$

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Worked example

$$f(x) = \ln\left(x + \sqrt{1 + x^2}\right)$$

(a) Show that

$$(1 + x^2)f'''(x) + 3xf''(x) + f'(x) = 0$$

(b) Deduce the values of $f'(0), f''(0), f'''(0)$

Your turn

$$f(x) = e^{x^2}$$

(a) Show that:

(i) $f'(x) = 2x\, f(x)$

(ii) $f''(x) = 2f(x) + 2x\, f'(x)$

(iii) $f'''(x) = 2xf''(x) + 4f'(x)$

(b) Deduce the values of $f'(0), f''(0), f'''(0)$

(a) Shown

(b) $f(0) = 1$

$f'(0) = 0$

$f''(0) = 2$

$f'''(0) = 0$