2A Method of Differences

1.

a) Show that:

$$4r^3 = r^2(r+1)^2 - (r-1)^2r^2$$

b) Hence, prove using the method of differences that:

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Verify that

$$\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$$

And hence find the following using the method of differences:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)}$$

3. Find the following summation using the method of differences:

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$$

4.

a) Express the following using partial fractions:

$$\frac{2}{(r+1)(r+3)}$$

b) Hence prove, by the method of differences, that:

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(an+b)}{6(n+2)(n+3)}$$

Where a and b are constants to be found.

c) Find the value of the following to 5 decimal places:

$$\sum_{r=21}^{30} \frac{2}{(r+1)(r+3)}$$