**2A Method of Differences**

1. Show that:

$$4r^{3}=r^{2}(r+1)^{2}-(r-1)^{2}r^{2}$$

1. Hence, prove using the method of differences that:

$$\sum\_{r=1}^{n}r^{3}=\frac{1}{4}n^{2}(n+1)^{2}$$

1. Verify that

$$\frac{1}{r(r+1)}=\frac{1}{r}-\frac{1}{r+1}$$

And hence find the following using the method of differences:

$$\sum\_{r=1}^{n}  \frac{1}{r(r+1)}$$

1. Find the following summation using the method of differences:

$$\sum\_{r=1}^{n}  \frac{1}{4r^{2}-1}$$

1. Express the following using partial fractions:

$$\frac{2}{(r+1)(r+3)}$$

1. Hence prove, by the method of differences, that:

$$\sum\_{r=1}^{n}\frac{2}{(r+1)(r+3)}= \frac{n(an+b)}{6(n+2)(n+3)}$$

Where a and b are constants to be found.

1. Find the value of the following to 5 decimal places:

$$\sum\_{r=21}^{30}\frac{2}{(r+1)(r+3)}$$