2.1) The method of differences

Worked example

Show that $r = \frac{1}{2}(r(r+1) - r(r-1))$

Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

Your turn

Show that
$$4r^3 = r^2(r+1)^2 - (r-1)^2 r^2$$

Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Shown

Worked example

Find, using the method of differences,

$$\sum_{r=1}^{n} \frac{1}{(r+2)(r+3)}$$

Your turn

Find, using the method of differences,

$$\sum_{r=1}^{n} \frac{1}{r(r+1)}$$

$$\frac{n}{n+1}$$

Worked example	Your turn
Find, using the method of differences, $$\sum_{r=1}^{n} \frac{2}{r(r+2)}$$	Find, using the method of differences, $$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)}$$ $$\frac{n(5n+13)}{6(n+2)(n+3)}$$

Worked example	Your turn
Find, using the method of differences, $$\sum_{r=1}^{n} \frac{2}{(4r^2 + 8r + 3)}$$	Find, using the method of differences, $$\sum_{r=1}^{n} \frac{2}{4r^2 - 1}$$ $$\frac{2n}{2n + 1}$$

Worked example	Your turn
Find the value of $$\sum_{r=100}^{200} \frac{4}{(4r-1)(4r+3)}$$ to 4 decimal places	Find the value of $$\sum_{r=16}^{25} \frac{4}{(2r+1)(2r+5)}$$ to 4 decimal places 0.0218 (4 dp)