2A Method of Differences

1.

a) Show that:

$$4r^3 = r^2(r+1)^2 - (r-1)^2 r^2$$

b) Hence, prove using the method of differences that:

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

2. Verify that

$$\frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$$

And hence find the following using the method of differences:

$$\sum_{r=1}^{n} \frac{1}{r(r+1)}$$

3. Find the following summation using the method of differences:

$$\sum_{r=1}^{n} \frac{1}{4r^2 - 1}$$

4.

a) Express the following using partial fractions:

$$\frac{2}{(r+1)(r+3)}$$

b) Hence prove, by the method of differences, that:

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+3)} = \frac{n(an+b)}{6(n+2)(n+3)}$$

Where a and b are constants to be found.

c) Find the value of the following to 5 decimal places:

$$\sum_{r=21}^{30} \frac{2}{(r+1)(r+3)}$$

2B Higher Derivatives for Maclaurin Series

1. Given that:

$$y = ln(1 - x)$$

Find the value of:

$$\left(\frac{d^3y}{dx^3}\right)_{\frac{1}{2}}$$

2. Given that:

$$f(x) = e^{x^2}$$

a) Show that:

$$f'(x) = 2xf(x)$$

b) By differentiating the result twice more with respect to x, find f''(x) and f'''(x)

c) Deduce the values of f(0), f'(0), f''(0) and f'''(0)

2C Maclaurin Series

1. Given that f(x) = e^x can be written in the form:

$$e^x = a_0 + a_1x + a_2x^2 + a_3x^3 + \ . \ . \ . \ + \ a_rx^r$$

And that it is valid to differentiate an infinite series term by term, show that:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ \dots \ + \frac{x^r}{r!}$$

Generalising:

2.

a) Express ln(1 + x) as an infinite series in ascending powers of x, up to and including the term in x^3

b) Using this series, find approximate values for:

i) ln(1.05)

ii) ln(1.25)

iii) ln(1.8)

3. Find the Maclaurin expansion for sinx, up to the term in x^5. Then use your expansion to find an approximation for sin10˚.

4. Find the Maclaurin expansion for cosx, up to the term in x^4.

5. Proving Euler's relation:

$$e^{i\theta} = cos\theta + isin\theta$$

2D Composite Maclaurin Series

Formula Book Formulae:

Maclaurin's and Taylor's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^r}{r!} f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leqslant 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \leqslant x \leqslant 1)$$

1. Write down the first 4 non-zero terms in the series expansion of $\cos(2x^2)$

2. Find the first 4 non-zero terms in the series expansion of:

$$ln\left\{\frac{\sqrt{1+2x}}{1-3x}\right\}$$

3. Given that terms in x^n, $n > 4$ can be ignored, show, using the series expansions of e^x and sinx, that:

$$e^{sinx} \approx 1 + x + \frac{x^2}{2} - \frac{x^4}{8}$$