**2A Method of Differences**

1. Show that:

$$4r^{3}=r^{2}(r+1)^{2}-(r-1)^{2}r^{2}$$

1. Hence, prove using the method of differences that:

$$\sum\_{r=1}^{n}r^{3}=\frac{1}{4}n^{2}(n+1)^{2}$$

1. Verify that

$$\frac{1}{r(r+1)}=\frac{1}{r}-\frac{1}{r+1}$$

And hence find the following using the method of differences:

$$\sum\_{r=1}^{n}  \frac{1}{r(r+1)}$$

1. Find the following summation using the method of differences:

$$\sum\_{r=1}^{n}  \frac{1}{4r^{2}-1}$$

1. Express the following using partial fractions:

$$\frac{2}{(r+1)(r+3)}$$

1. Hence prove, by the method of differences, that:

$$\sum\_{r=1}^{n}\frac{2}{(r+1)(r+3)}= \frac{n(an+b)}{6(n+2)(n+3)}$$

Where a and b are constants to be found.

1. Find the value of the following to 5 decimal places:

$$\sum\_{r=21}^{30}\frac{2}{(r+1)(r+3)}$$

**2B Higher Derivatives for Maclaurin Series**

1. Given that:

$$y=ln\left(1-x\right)$$

Find the value of:

$$\left(\frac{d^{3}y}{dx^{3}}\right)\_{\frac{1}{2}}$$

1. Given that:

$$f\left(x\right)=e^{x^{2}}$$

1. Show that:

$$f'\left(x\right)=2xf(x)$$

1. By differentiating the result twice more with respect to x, find f’’(x) and f’’’(x)
2. Deduce the values of f(0), f’(0), f’’(0) and f’’’(0)

**2C Maclaurin Series**

1. Given that f(x) = ex can be written in the form:

$$e^{x}=a\_{0}+a\_{1}x+a\_{2}x^{2}+a\_{3}x^{3}+ .. . + a\_{r}x^{r}$$

And that it is valid to differentiate an infinite series term by term, show that:

$$e^{x}=1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+ … +\frac{x^{r}}{r!} $$

Generalising:

1. Express ln(1 + x) as an infinite series in ascending powers of x, up to and including the term in x3
2. Using this series, find approximate values for:
3. ln(1.05)
4. ln(1.25)
5. ln(1.8)
6. Find the Maclaurin expansion for sinx, up to the term in x5. Then use your expansion to find an approximation for sin10˚.
7. Find the Maclaurin expansion for cosx, up to the term in x4.
8. Proving Euler’s relation:

$$e^{iθ}=cosθ+isinθ$$

**2D Composite Maclaurin Series**

Formula Book Formulae:



1. Write down the first 4 non-zero terms in the series expansion of cos(2x2)
2. Find the first 4 non-zero terms in the series expansion of:

$$ln\left\{\frac{\sqrt{1+2x}}{1-3x}\right\}$$

1. Given that terms in xn, n > 4 can be ignored, show, using the series expansions of ex and sinx, that:

$$e^{sinx}≈1+x+\frac{x^{2}}{2}-\frac{x^{4}}{8}$$