Applications of De’Moivre’s Theorem 2: Roots

De Moivre’s theorem also holds true for rational powers. We can use De Moivre’s to solve equations of the form , where . This is equivalent to finding the nth roots of

The fundamental theorem of algebra holds true for complex numbers:

Hence , where has n distinct roots.

If w = 1 we call these roots of unity.

To find the roots of a complex equation we use the fact that the argument of a complex number is not unique:

If then )

**Roots of Unity Example**

**Solve**

Method 1: By factorising Method 2: De Moivre’s

Notice:

* The first root will always be 1 since
* We add to the argument but leave the modulus unchanged e.g. when we rotate the line each time. When we have rotated so we get back to where we started.
* The first root is z1 = 1, call the second root z2 = . Then…

Consider

What do you notice?

What would z3 be?

The roots of can be represented as

Since the resultant ‘vector’ is 0, then

These roots form the vertices of a regular n-gon and all lie on a circle, radius r.

General nth Roots:

We can use a similar method when w is not equal to 1. Again, our first step is to write w in mod-arg form and consider multiples of the argument.

**Example**

**Solve**

Test your understanding

1. Express the complex number in the form , . (3)
2. Solve the equation  
   giving the roots in the form , . (5)

Ex 1F pg 24