1.6) nth roots of a complex number

Worked example	Your turn
Solve $$z^8 = 1$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $$z^4 = 1$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = i$ $z_3 = -1$ $z_4 = -i$

Worked example	Your turn
Solve $$z^7 - 1 = 0$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $$z^5 - 1 = 0$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = 0.309 + 0.951i$ $z_3 = -0.809 + 0.588i$ $z_4 = -0.809 - 0.588i$ $z_5 = 0.309 - 0.951i$

Worked example	Your turn
Solve $z^3 = -1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $z^3 = 1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $z_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Worked example	Your turn
Find the cubic roots of unity, and the value of their sum.	Find the quintic roots of unity, and the value of their sum.

$1, e^{\frac{2\pi}{5}i}, e^{\frac{4\pi}{5}i}, e^{\frac{-4\pi}{5}i}, e^{\frac{-2\pi}{5}i}$

$$z^5 - 1 = 0$$
$$(z-1)(z^4 + z^3 + z^2 + z + 1) = 0$$

$z = e^{\frac{2\pi}{5}i}$, so $z - 1 \neq 0$

$\therefore z^4 + z^3 + z^2 + z + 1 = 0$

Worked example	Your turn
Solve $z^3 = -1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ Represent your solutions on an Argand diagram	Solve $z^3 = 1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ Represent your solutions on an Argand diagram

Worked example	Your turn
Solve $z^4 = -2 + 2\sqrt{3}\,i$ Express the roots in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \le \pi$	Solve $z^4 = 2 + 2\sqrt{3}\,i$ Express the roots in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \le \pi$ $z_1 = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ $z_2 = \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ $z_3 = \sqrt{2}\left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right)$ $z_4 = \sqrt{2}\left(\cos\left(-\frac{11\pi}{12}\right) + i\sin\left(-\frac{11\pi}{12}\right)\right)$

Worked example	Your turn
Solve $z^3 + 32\sqrt{2} + 32i\sqrt{2} = 0$ Express the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$	Solve $z^3 + 4\sqrt{2} + 4i\sqrt{2} = 0$ Express the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$ $z_1 = 2e^{-\frac{\pi i}{4}}$ $z_2 = 2e^{\frac{5\pi i}{12}}$ $z_3 = 2e^{-\frac{11\pi i}{12}}$

Worked example	Your turn
Find the three roots of the equation $(z-1)^3 = -1$ Plot the points representing these three roots on an Argand diagram. Given that these three points lie on a circle, find its centre and radius	Find the three roots of the equation $(z+1)^3 = -1$ Plot the points representing these three roots on an Argand diagram. Given that these three points lie on a circle, find its centre and radius Centre (1,0) ; Radius 1