

1.5) Solving cubic and quartic equations

Worked example

Given that -2 is a root of the cubic equation

$$z^3 - 2z^2 - 3z + k = 0$$

- (a) Find the value of k
- (b) Find the other two roots

Your turn

Given that -1 is a root of the cubic equation

$$z^3 - z^2 + 3z + k = 0$$

- (a) Find the value of k
- (b) Find the other two roots

(a) $k = 5$

(b) $1 + 2i$ and $1 - 2i$

Worked example

Given that $3 + i$ is a root of the quartic equation

$2z^4 - 37z^3 + 221z^2 - 380z - 250 = 0$,
solve the equation completely.

Your turn

Given that $3 + i$ is a root of the quartic equation

$2z^4 - 3z^3 - 39z^2 + 120z - 50 = 0$, solve
the equation completely.

$$z_1 = -5$$

$$z_2 = \frac{1}{2}$$

$$z_3 = 3 + i$$

$$z_4 = 3 - i$$

Worked example

Show that $z^2 + 9$ is a factor of

$$z^4 - 8z^3 + 26z^2 - 72z + 153$$

Hence solve the equation

$$z^4 - 8z^3 + 26z^2 - 72z + 153 = 0$$

Your turn

Show that $z^2 + 4$ is a factor of

$$z^4 - 2z^3 + 21z^2 - 8z + 68$$

Hence solve the equation

$$z^4 - 2z^3 + 21z^2 - 8z + 68 = 0$$

$$z_1 = 2i$$

$$z_2 = -2i$$

$$z_3 = 1 + 4i$$

$$z_4 = 1 - 4i$$

Worked example

Given that 5 and $4 + 3i$ are roots of the equation

$$x^3 - 13x^2 + cx + d = 0 \quad c, d \in \mathbb{R}$$

- (a) Write down the other complex root
- (b) Find the value of c and the value of d

Your turn

Given that 2 and $5 + 2i$ are roots of the equation

$$x^3 - 12x^2 + cx + d = 0 \quad c, d \in \mathbb{R}$$

- (a) Write down the other complex root
- (b) Find the value of c and the value of d

(a) $5 - 2i$

(b) $c = 49, d = -58$

Worked example

Solve:

$$z^4 = 1$$

$$z^4 = 16$$

Your turn

Solve:

$$z^4 = 81$$

$$z_1 = 3$$

$$z_2 = -3$$

$$z_3 = 3i$$

$$z_4 = 1 - 4i$$

Worked example

$$f(z) = z^3 + 4z^2 + kz + 36, k \in \mathbb{R}$$

Given that $f(3i) = 0$, find the value of k and the other two roots of the equation

Your turn

$$f(z) = z^3 + 3z^2 + kz + 48, k \in \mathbb{R}$$

Given that $f(4i) = 0$, find the value of k and the other two roots of the equation

$$k = 16$$
$$-4i \text{ and } -3$$

Worked example

Find the square root of $3 + 4i$

Find the square root of i

Your turn

Find the square root of $5 + 12i$

$$3 - 2i, -3 + 2i$$