

Example

(i) Show that $(2 + e^{i\theta})(2 + e^{-i\theta}) = 5 + 4 \cos \theta$.

(ii) Let $S = \frac{\sin \theta}{2} - \frac{\sin 2\theta}{2^2} + \frac{\sin 3\theta}{2^3} - \frac{\sin 4\theta}{2^4} + \dots$

By considering $C - iS$ where $C = 1 - \frac{\cos \theta}{2} + \frac{\cos 2\theta}{2^2} - \frac{\cos 3\theta}{2^3} + \frac{\cos 4\theta}{2^4} - \dots$,

show that $S = \frac{2 \sin \theta}{5 + 4 \cos \theta}$.