

1.5) Sums of series

Worked example

$$\frac{5}{e^{2i\theta} - 1}$$

$$\frac{2e^{i\theta}}{e^{6i\theta} - 1}$$

$$\frac{7}{\frac{i\theta}{e^{\frac{i\theta}{5}} - 1}}$$

Your turn

$$\frac{3e^{i\theta}}{e^{4i\theta} - 1}$$

$$\frac{3e^{-i\theta}}{2i \sin 2\theta}$$

$$\frac{11}{\frac{i\theta}{e^{\frac{i\theta}{3}} - 1}}$$

$$\frac{11e^{-\frac{i\theta}{6}}}{2i \sin \frac{\theta}{6}}$$

Worked example

$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \dots + e^{6i\theta}$, for $\theta \neq 2n\pi$,
where n is an integer.

a) Show that $S = \frac{e^{\frac{7i\theta}{2}} \sin 3\theta}{\sin \frac{\theta}{2}}$

Your turn

$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \dots + e^{8i\theta}$, for $\theta \neq 2n\pi$,
where n is an integer.

a) Show that $S = \frac{e^{\frac{9i\theta}{2}} \sin 4\theta}{\sin \frac{\theta}{2}}$

Shown

Worked example

$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \dots + e^{6i\theta}$, for $\theta \neq 2n\pi$,
where n is an integer.

a) Show that $S = \frac{e^{\frac{7i\theta}{2}} \sin 3\theta}{\sin \frac{\theta}{2}}$

Let $P = \cos \theta + \cos 2\theta + \cos 3\theta + \dots + \cos 6\theta$ and
 $Q = \sin \theta + \sin 2\theta + \dots + \sin 6\theta$

(b) Use your answer to part a to show that
 $P = \cos \frac{7\theta}{2} \sin 3\theta \operatorname{cosec} \frac{\theta}{2}$ and find similar
expressions for Q and $\frac{Q}{P}$

Your turn

$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \dots + e^{8i\theta}$, for $\theta \neq 2n\pi$,
where n is an integer.

a) Show that $S = \frac{e^{\frac{9i\theta}{2}} \sin 4\theta}{\sin \frac{\theta}{2}}$

Let $P = \cos \theta + \cos 2\theta + \cos 3\theta + \dots + \cos 8\theta$ and
 $Q = \sin \theta + \sin 2\theta + \dots + \sin 8\theta$

(b) Use your answer to part a to show that
 $P = \cos \frac{9\theta}{2} \sin 4\theta \operatorname{cosec} \frac{\theta}{2}$ and find similar
expressions for Q and $\frac{Q}{P}$

Shown

$$Q = \sin \frac{9\theta}{2} \sin 4\theta \operatorname{cosec} \frac{\theta}{2}$$
$$\frac{Q}{P} = \tan \frac{9\theta}{2}$$