Applications of de Moivre's

Trig identities

De Moivre's theorem can be used to give multiple angle expressions ($\cos n\theta$ / $\sin n\theta$) in terms of powers, and to express powers of sin and cos in terms of multiple angles. This is useful in integration.

We derive these identities by applying the binomial expansion to $(\cos\theta + isin\theta)^n$

Recap

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{(n-2)}b^2$+….

a) Expressing $\cos n\theta$ and $\sin n\theta$ in terms of powers of $\cos\theta$

Example

Express $\cos 3\theta$ in terms of powers of $\cos\theta$

Test Your understanding

1. **Express $\cos 6\theta$ in terms of $\cos \theta$**

2. (a) Use de Moivre's theorem to show that **(5)**

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

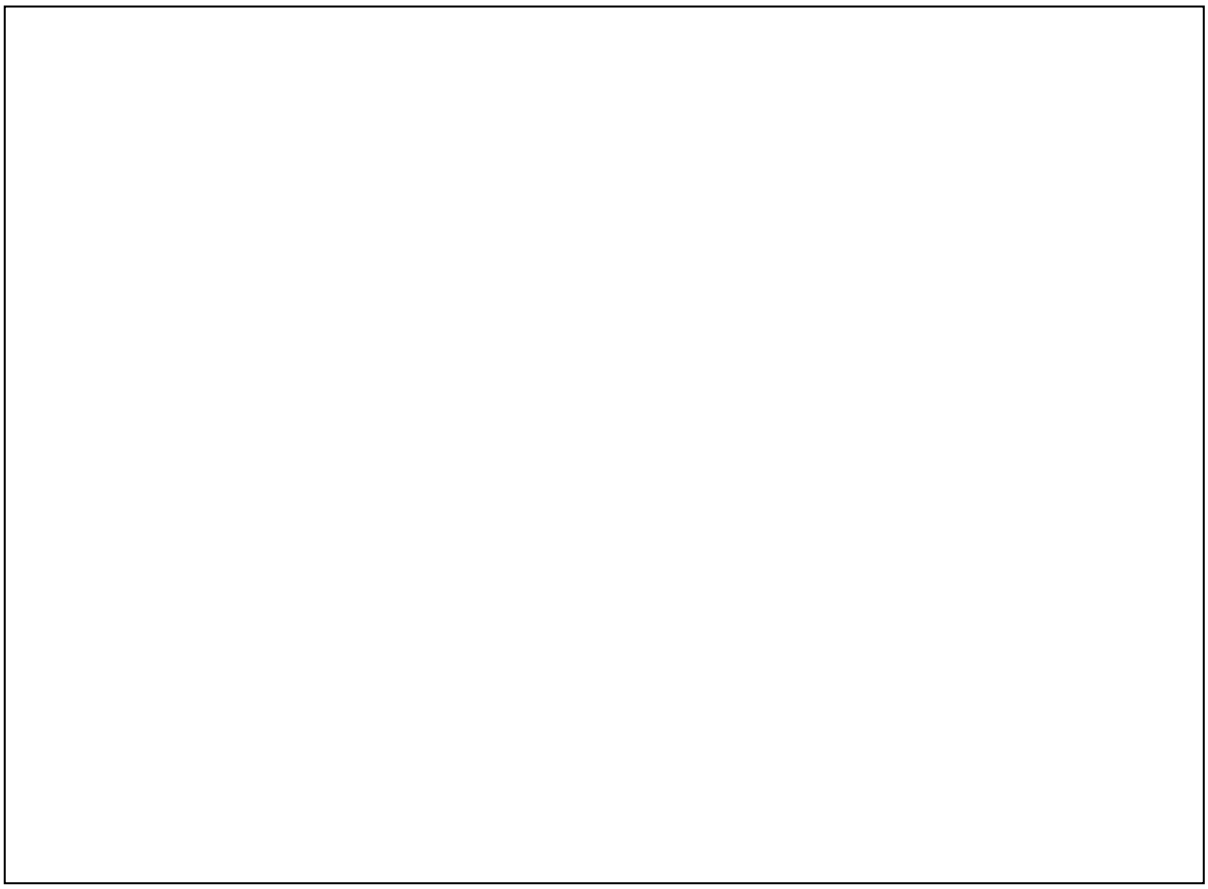
Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(b) Find all the solutions of

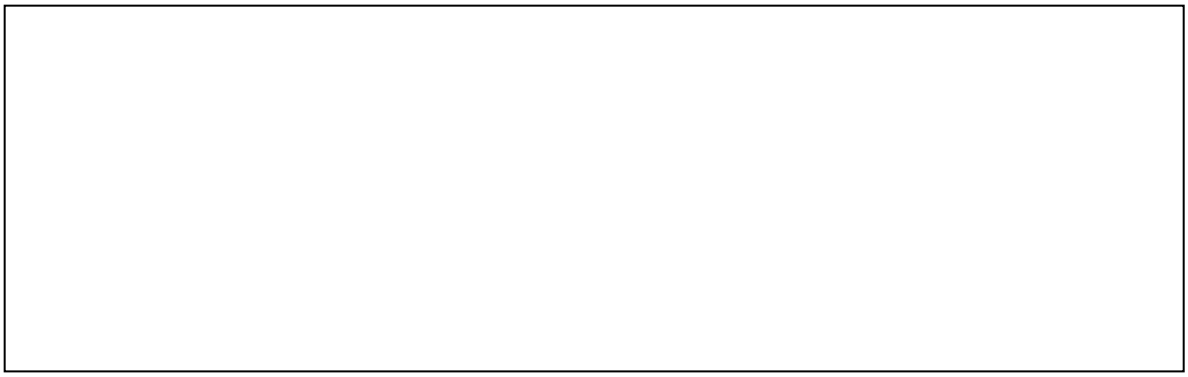
$$\sin 5\theta = 5\sin 3\theta$$

in the interval $0 \leq \theta < 2\pi$. Give your answers to 3 decimal places. **(6)**

b. Finding expressions for $\sin^n \theta$ and $\cos^n \theta$



Exponential Form



Examples

1. Express $\cos^5\theta$ in the form $a\cos 5\theta + b\cos 3\theta + c\cos\theta$

2. Prove that $\sin^3\theta = -\frac{1}{4}\sin 3\theta + \frac{3}{4}\sin\theta$

Test Your Understanding

a) Express $\sin^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$

(b) Hence find the exact value of $\int_0^{\frac{\pi}{2}} \sin^4\theta \ d\theta$