1.4) Trigonometric identities

Worked example

Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

Your turn

Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Shown

Worked example

Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

Your turn

Use de Moivre's theorem to show that

$$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$$

Shown

Worked example

Use de Moivre's theorem to show that

$$\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$$

Your turn

Use de Moivre's theorem to show that

$$\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$$

Shown

Worked example	Your turn
Use de Moivre's theorem to show that $\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$	Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$ Shown

Worked example

Use de Moivre's theorem to show that

$$\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$$

Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin^5\theta$$

Your turn

Use de Moivre's theorem to show that

$$\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$$

Hence find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin^4\theta$$

$$\frac{3\pi}{16}$$

Worked example	Your turn
Show that $32\cos^2\theta\sin^4\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$	Show that $32\sin^2\theta\cos^4\theta = -\cos 6\theta - 2\cos 4\theta + \cos 2\theta + 2$ Shown

Worked example	Your turn
Show that $32\cos^2\theta\sin^4\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$ Hence, find the exact value of $$\int_0^{\frac{\pi}{3}} \cos^2\theta\sin^4\theta$$	Show that $32\sin^2\theta\cos^4\theta = -\cos 6\theta - 2\cos 4\theta + \cos 2\theta + 2$ Hence, find the exact value of $$\int_0^{\frac{\pi}{4}} \sin^2\theta\cos^4\theta$$ $$\frac{\pi}{64} + \frac{1}{48}$$

Worked example	Your turn
Use de Moivre's theorem to show that $$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$ Hence find the distinct solutions of the equation $16x^5 - 20x^3 + 5x - \frac{1}{2} = 0$, giving your answers to 3 decimal places where necessary.	Use de Moivre's theorem to show that $$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$ Hence find the six distinct solutions of the equation $32x^6 - 48x^4 + 18x^2 - \frac{3}{2} = 0$, giving your answers to 3 decimal places where necessary. $$x = \pm 0.342, \pm 0.643, \pm 0.985$$

Worked example

Use de Moivre's theorem to show that

$$\cos 5\theta = \cos\theta\,(16\cos^4\theta - 20\cos^2\theta + 5)$$

Hence solve for $0 \le \theta < \pi$

$$\cos 5\theta - \cos\theta\cos 2\theta = 0$$

Your turn

Use de Moivre's theorem to show that

$$\sin 5\theta = \sin\theta\,(16\cos^4\theta - 12\cos^2\theta + 1)$$

Hence solve for $0 \le \theta < \pi$

$$\sin 5\theta + \cos\theta\sin 2\theta = 0$$

$\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}$, 1.209 (3 dp) and 1.932 (3 dp)