1.3) de Moivre's theorem

Worked example	Your turn
Use de Moivre's theorem to express in the form $x + iy$, where $x, y \in \mathbb{R}$	Use de Moivre's theorem to express in the form $x + iy$, where $x, y \in \mathbb{R}$
$(\cos\theta + i\sin\theta)^5$	$(\cos\theta + i\sin\theta)^7$
	$\cos 7\theta + i\sin 7\theta$
$(\cos 2\theta + i\sin 2\theta)^3$	$(\cos 3\theta + i\sin 3\theta)^5$
	$\cos 15\theta + i\sin 15\theta$

Worked example	Your turn
Express in the form $e^{ni\theta}$ $$\frac{(\cos 5\theta + i\sin 5\theta)^3}{(\cos 3\theta + i\sin 3\theta)^7}$$ $$\frac{(\cos 2\theta + i\sin 2\theta)^5}{(\cos 7\theta - i\sin 7\theta)^3}$$	Express in the form $e^{ni\theta}$ $$\frac{(\cos 3\theta + i\sin 3\theta)^7}{(\cos 5\theta + i\sin 5\theta)^4}$$ $$e^{i\theta}$$

Worked example	Your turn
Simplify $$\frac{\left(\cos\frac{3\pi}{11}+i\sin\frac{3\pi}{11}\right)^2}{\left(\cos\frac{2\pi}{11}-i\sin\frac{2\pi}{11}\right)^{19}}$$	Simplify $$\frac{\left(\cos\frac{9\pi}{17}+i\sin\frac{9\pi}{17}\right)^5}{\left(\cos\frac{2\pi}{17}-i\sin\frac{2\pi}{17}\right)^3}$$ -1

Worked example	Your turn
Express in the form $x + iy$ where $x, y \in \mathbb{R}$ $$\left(1 - \sqrt{3}i\right)^6$$	Express in the form $x + iy$ where $x, y \in \mathbb{R}$ $$\left(1 + \sqrt{3}\, i\right)^7$$ $$64 + 64\sqrt{3}\, i$$

Worked example	Your turn
$w = \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$, find the exact value of: w^6	$z = \sqrt{5}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ Giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$, find the exact value of: z^4 $-\frac{25}{2} + \frac{25\sqrt{3}}{2}i$

Worked example	Your turn
$w = -2 - 2\sqrt{3}i$ Using de Moivre's Theorem, find w^5 w^4	$z = -8 + 8\sqrt{3}i$ Using de Moivre's Theorem, find z^3 4096

Worked example	Your turn
Use de Moivre's theorem to show that $(a+bi)^n-(a-bi)^n$ is imaginary for all integers n	Use de Moivre's theorem to show that $(a+bi)^n+(a-bi)^n$ is real for all integers n

$$(a+bi)^n+(a-bi)^n$$
$$=\left(re^{i\theta}\right)^n-\left(re^{-i\theta}\right)^n$$
$$=re^{in\theta}-re^{-in\theta}$$
$$=r^n\left(e^{in\theta}-e^{-in\theta}\right)$$
$$=r^n(\cos n\theta+i\sin n\theta-(\cos n\theta-i\sin n\theta)$$
$$=r^n(2i\sin n\theta)$$
$$=2r^n\sin n\theta\ (i)$$

Worked example	Your turn
Using Euler's relation, prove that if n is a positive integer, $\left(r(\cos\theta + i\sin\theta)\right)^n = r^n(\cos n\theta + i\sin n\theta)$	Using Euler's relation, prove that if n is a positive integer, $\left(r(\cos\theta + i\sin\theta)\right)^{-n} = r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$

$$\left(r(\cos\theta + i\sin\theta)\right)^{-n}$$

$$= (re^{i\theta})^{-n}$$

$$= r^{-n}e^{-in\theta}$$

$$= r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$

Worked example	Your turn
Without using Euler's relation, prove that if n is a positive integer, $$\left(r(\cos\theta + i\sin\theta)\right)^n = r^n(\cos n\theta + i\sin n\theta)$$	Without using Euler's relation, prove that if n is a positive integer, $$\left(r(\cos\theta + i\sin\theta)\right)^{-n} = r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$

$$\left(r(\cos\theta + i\sin\theta)\right)^{-n}$$

$$= \frac{1}{\left(r(\cos\theta + i\sin\theta)\right)^n}$$

$$= \frac{1}{r^n(\cos n\theta + i\sin n\theta)}$$

$$= \frac{1}{r^n(\cos n\theta + i\sin n\theta)} \times \frac{\cos n\theta - i\sin n\theta}{\cos n\theta - i\sin n\theta}$$

$$= \frac{\cos n\theta - i\sin n\theta}{r^n(\cos^2 n\theta - i^2\sin^2 n\theta)}$$

$$= \frac{\cos n\theta - i\sin n\theta}{r^n(\cos^2 n\theta + \sin^2 n\theta)}$$

$$= r^{-n}(\cos n\theta - i\sin n\theta)$$

$$= r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$