

1.2) Multiplying and dividing complex numbers

Worked example

$$2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \times 5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

Your turn

$$3 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \times 4 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$$

12i

Worked example

$$2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) \times 5 \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)$$

Your turn

$$3 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \times 4 \left(\cos \frac{\pi}{12} - i \sin \frac{\pi}{12} \right)$$

$$6 + 6i\sqrt{3}$$

Worked example

$$\frac{2 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)}{5 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)}$$

Your turn

$$\frac{3 \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)}{4 \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)}$$

$$\frac{3}{8} + \frac{3\sqrt{3}}{8}i$$

Worked example

Express in the form $x + iy$

$$3e^{\frac{\pi i}{12}} \times \sqrt{2}e^{\frac{\pi i}{4}}$$

$$5e^{\frac{\pi i}{4}} \times \sqrt{7}e^{\frac{\pi i}{2}}$$

Your turn

Express in the form $x + iy$

$$2e^{\frac{\pi i}{6}} \times \sqrt{3}e^{\frac{\pi i}{3}}$$

$$2i\sqrt{3}$$

Worked example

Express in the form $x + iy$

$$\frac{3e^{\frac{\pi i}{4}}}{6e^{\frac{\pi i}{12}}}$$

$$\frac{5e^{\frac{\pi i}{4}}}{\sqrt{7}e^{-\frac{\pi i}{2}}}$$

Your turn

Express in the form $x + iy$

$$\frac{\sqrt{5}e^{-\frac{\pi i}{4}}}{7e^{\frac{\pi i}{2}}}$$

$$-\frac{\sqrt{10}}{14} - \frac{\sqrt{10}}{14}i$$

Worked example

$$z = 1 - i$$

Find

(a) $|z|$

(b) $\arg(z)$ in terms of π

$$w = 3 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$$

Find

(c) $\left| \frac{w}{z} \right|$

(d) $\arg \left| \frac{w}{z} \right|$

Your turn

$$z = 5\sqrt{3} - 5i$$

Find

(a) $|z|$

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(b) $\arg(z)$ in terms of π

 $-\frac{\pi}{6}$

$$w = 2 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

Find

(c) $\left| \frac{w}{z} \right|$

 $\frac{1}{5}$

(d) $\arg \left| \frac{w}{z} \right|$

 $\frac{5\pi}{12}$

Worked example

$$z = 3 + 3i$$

$$\operatorname{Im}(zw) = 0$$

$$|zw| = 2|z|$$

Use geometrical reasoning to find the two possibilities for w , giving your answers in exponential form

Your turn

$$z = 2 + 2i$$

$$\operatorname{Im}(zw) = 0$$

$$|zw| = 3|z|$$

Use geometrical reasoning to find the two possibilities for w , giving your answers in exponential form

$$w_1 = 3e^{-\frac{\pi i}{4}}, w_2 = 3e^{\frac{3\pi i}{4}}$$