

1.1) Imaginary and complex numbers

Worked example

Write in terms of i :

$$\sqrt{-39}$$

$$\sqrt{-40}$$

Your turn

Write in terms of i :

$$\sqrt{-49}$$

$$7i$$

$$\sqrt{-20}$$

$$(2\sqrt{5})i$$

Worked example

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$(2 + 5i) + (3 + 4i)$$

$$(2 - 5i) - (4 - 3i)$$

Your turn

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$(2 + 3i) + (4 + 5i)$$

$$6 + 8i$$

$$(2 - 3i) - (4 - 5i)$$

$$-2 + 2i$$

Worked example

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$2(3 + 4i)$$

$$-5(6 - 7i)$$

Your turn

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$-8(9 + 10i)$$

$$-72 - 80i$$

Worked example

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$\frac{6 - 8i}{2}$$

$$\frac{-7 + 21i}{7}$$

Your turn

Simplify, giving your answers in the form $a + bi$, where $a, b \in \mathbb{R}$:

$$\frac{15 - 12i}{3}$$

$$5 - 4i$$

Worked example

Given that $z_1 = a + 2i$, $z_2 = -3 + bi$, and $z_2 - z_1 = 5 + 7i$, find a and b , where $a, b \in \mathbb{R}$

Your turn

Given that $z_1 = a + 5i$, $z_2 = -2 + 7i$, and $z_2 - z_1 = 3 + 11i$, find a and b , where $a, b \in \mathbb{R}$

$$a = -5, b = 16$$

Worked example

Given that $z = a + bi$, and $w = a - bi$,
where $a, b \in \mathbb{R}$, show that:

$z + w$ is always real

Your turn

Given that $z = a + bi$, and $w = a - bi$,
where $a, b \in \mathbb{R}$, show that:

$z - w$ is always imaginary

$$\begin{aligned} & (a + bi) - (a - bi) \\ &= a + bi - a + bi \\ &= 2bi \\ &= (2b)i \end{aligned}$$

Worked example

Solve:

$$z^2 = -9$$

$$z^2 + 16 = 0$$

Your turn

Solve:

$$z^2 + 25 = 0$$

$$z = \pm 5i$$

Worked example

Solve:

$$(z + 2)^2 + 9 = 0$$

$$(z - 3)^2 + 16 = 0$$

Your turn

Solve:

$$(z + 4)^2 + 25 = 0$$

$$z = -4 \pm 5i$$

Worked example

Solve:

$$z^2 + 4z + 13 = 0$$

$$z^2 - 6z + 25 = 0$$

Your turn

Solve:

$$z^2 + 8z + 41 = 0$$

$$z = -4 \pm 5i$$

Worked example

Solve:

$$z^2 + 3z + 13 = 0$$

$$3z^2 - 7z + 25 = 0$$

Your turn

Solve:

$$2z^2 - 8z + 41 = 0$$

$$z = 2 \pm \frac{\sqrt{66}}{2}i$$

Worked example

The equation $z^2 + bz + 31 = 0$, where $b \in \mathbb{R}$, has distinct, non-real complex roots. Find the range of possible values of b

Your turn

The equation $z^2 + bz + 13 = 0$, where $b \in \mathbb{R}$, has distinct, non-real complex roots. Find the range of possible values of b

$$-2\sqrt{13} < b < 2\sqrt{13}$$