

1.1) Exponential form of complex numbers

Worked example

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$3 - 2i$$

$$1$$

$$-i$$

Your turn

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$2 - 3i$$

$$\sqrt{13}e^{-0.983i}$$

$$-1$$

$$e^{i\pi}$$

$$i$$

$$e^{\frac{i\pi}{2}}$$

Worked example

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$\sqrt{2} \left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5} \right)$$

$$\sqrt{13}(\cos(0.983) + i \sin(0.983))$$

$$\cos \pi + i \sin \pi$$

Your turn

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$\sqrt{3} \left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right)$$

$$\sqrt{3}e^{\frac{\pi i}{8}}$$

$$\sqrt{13}(\cos(-0.983) + i \sin(-0.983))$$

$$\sqrt{13}e^{-0.983i}$$

$$\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

$$e^{\frac{i\pi}{2}}$$

Worked example

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$\sqrt{2} \left(\cos \frac{\pi}{5} - i \sin \frac{\pi}{5} \right)$$

$$\sqrt{3} \left(\cos \left(-\frac{\pi}{7} \right) - i \sin \left(-\frac{\pi}{7} \right) \right)$$

Your turn

Express in the form $re^{i\theta}$, where
 $-\pi < \theta \leq \pi$:

$$\sqrt{5} \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)$$

$$\sqrt{5} e^{-\frac{\pi i}{3}}$$

Worked example

Express in the form $x + iy$, where $x, y \in \mathbb{R}$:

$$\sqrt{2}e^{\frac{3\pi i}{4}}$$

$$e^{-\frac{i\pi}{2}}$$

Your turn

Express in the form $x + iy$, where $x, y \in \mathbb{R}$:

$$\sqrt{2}e^{-\frac{3\pi i}{4}}$$

$$z = -1 - i$$

$$e^{i(0)}$$

$$z = 1$$

Worked example

Express in the form
 $r(\cos \theta + i \sin \theta)$, where
 $-\pi < \theta \leq \pi$:

$$2e^{\frac{23\pi i}{5}}$$

$$3e^{\frac{-25\pi i}{8}}$$

Your turn

Express in the form
 $r(\cos \theta + i \sin \theta)$, where
 $-\pi < \theta \leq \pi$:

$$4e^{\frac{33\pi i}{7}}$$

$$4\left(\cos \frac{5\pi}{7} + i \sin \frac{5\pi}{7}\right)$$

Worked example

Use $e^{i\theta} = \cos \theta + i \sin \theta$ to show that $\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

Your turn

Use $e^{i\theta} = \cos \theta + i \sin \theta$ to show that $\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\begin{aligned} e^{-i\theta} &= \cos(-\theta) + i \sin(-\theta) \\ &= \cos \theta - i \sin \theta \end{aligned}$$

$$e^{i\theta} - e^{-i\theta} = 2i \sin \theta$$

$$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$