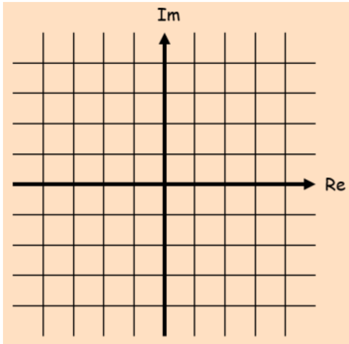
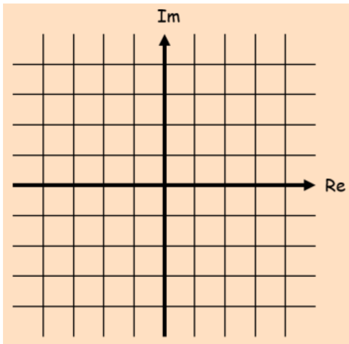
1A Exponential Form

1. Express the numbers following numbers in the modulus argument form:

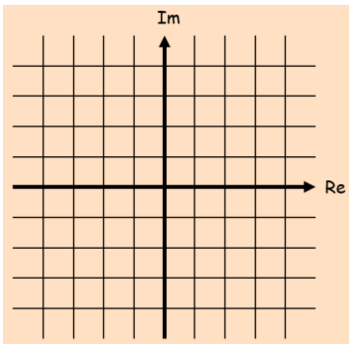
a) $z_1 = 1 + i\sqrt{3}$



b) $z_2 = -3 - 3i$



2. Express the complex number $\mathrm{z} = 2 - 3\mathrm{i}$ in the form $\mathrm{re}^{\mathrm{i}\theta}$, where $-\pi < \theta \leq \pi$



3. Express the following in the form $z = re^{i\theta}$ where $-\pi < \theta \leq \pi$

a) $z = \sqrt{2}\left(cos\left(\frac{\pi}{10}\right) + isin\left(\frac{\pi}{10}\right)\right)$

b) $z = 5\left(cos\left(\frac{\pi}{8}\right) - isin\left(\frac{\pi}{8}\right)\right)$

4. Express the following in the form $z = x + iy$ where $x \in \mathbb{R}$ and $y \in \mathbb{R}$

$$z = \sqrt{2}e^{\frac{3\pi}{4}i}$$

5. Express the following in the form $r(cos\theta + isin\theta)$, where $-\pi < \theta \leq \pi$

$$z = 2e^{\frac{23\pi}{5}i}$$

6. Use: $e^{i\theta} = cos\theta + isin\theta$ To show that: $cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$

1B Multiplying & Dividing Complex Numbers

1. Express the following calculation in the form x + iy:

$$\frac{\sqrt{2}\left(cos\frac{\pi}{12} + isin\frac{\pi}{12}\right)}{2\left(cos\frac{5\pi}{6} + isin\frac{5\pi}{6}\right)}$$

2. Express $2e^{\frac{\pi i}{6}} \times \sqrt{3}e^{\frac{\pi i}{3}}$ in the form $x + iy$

3. $z = 2 + 2i, Im(zw) = 0$ and $|zw| = 3|z|$

1C De Moivre's Theorem

1. Proof by Induction:

   Negatives:

   0:

1. Simplify:

$$\frac{\left(cos\left(\frac{9\pi}{17}\right)+isin\left(\frac{9\pi}{17}\right)\right)^5}{\left(cos\left(\frac{2\pi}{17}\right)-isin\left(\frac{2\pi}{17}\right)\right)^3}$$

2. Express $\left(1+i\sqrt{3}\right)^7$ in the form $x+iy$, where $x, y \in \mathbb{R}$

1D Using de Moivre to Prove Trigonometric Identities

1. Express $\cos 3\theta$ using powers of $\cos\theta$.

2. Use De Moivre's theorem to show that:

$$cos6\theta = 32cos^6\theta - 48cos^4\theta + 18cos^2\theta - 1$$

Further notes:

$$z + \frac{1}{z}$$

$$z - \frac{1}{z}$$

$$z^n + \frac{1}{z^n}$$

$$z^n - \frac{1}{z^n}$$

3. Express $\cos^5\theta$ in the form $\mathbf{a}\cos5\theta + \mathbf{b}\cos3\theta + \mathbf{c}\cos\theta$

   Where **a**, **b** and **c** are constants to be found.

4.

a) Express $\sin^4\theta$ in the form:

$$d\cos 4\theta + e\cos 2\theta + f$$

b) Hence, find the exact value of the following integral:

$$\int_0^{\frac{\pi}{2}} \sin^4\theta \; d\theta$$

1E Part 1 Finite Summations

1. Given that $z = cos\left(\frac{\pi}{n}\right) + isin\left(\frac{\pi}{n}\right)$, where n is a positive integer, show that:

$$1 + z + z^2 + .. + z^{n-1} = 1 + icot\left(\frac{\pi}{2n}\right)$$

Notes for $e^{i\theta}+e^{2i\theta}+e^{3i\theta}+..+e^{ni\theta}$

2. $S=e^{i\theta}+e^{2i\theta}+e^{3i\theta}+..+e^{8i\theta}$, for $\theta \neq 2n\pi$, where n is an integer

a) Show that

$$S=\frac{e^{\frac{9i\theta}{2}}sin4\theta}{sin\left(\frac{\theta}{2}\right)}$$

Let: $P = cos\theta + cos2\theta + cos3\theta + .. + cos8\theta \;\; and \;\; Q = sin\theta + sin2\theta + sin3\theta + .. + sin8\theta$

b) Use your answer to part a to show that $P = cos\frac{9\theta}{2} sin4\theta cosec\frac{\theta}{2}$, and find similar expressions for Q and $\frac{P}{Q}$

1E Part 2 Infinite Summations

1. $S = 1 + \frac{1}{2}e^{i\theta} + \frac{1}{4}e^{2i\theta} + \frac{1}{8}e^{3i\theta} + \cdots$

a) Show that

$$S = \frac{4 - 2\cos\theta + 2i\sin\theta}{5 - 4\cos\theta}$$

Let: $P = 1 + \frac{1}{2}cos\theta + \frac{1}{4}cos2\theta + \frac{1}{8}cos3\theta + \cdots \; and \; Q = \frac{1}{2}sin\theta + \frac{1}{4}sin2\theta + \frac{1}{8}sin3\theta + \cdots$

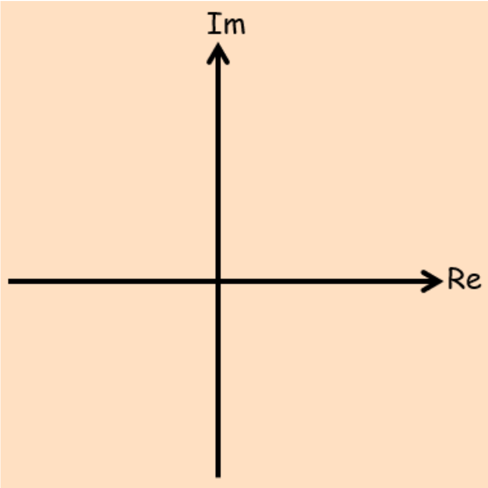
b) Show that $S = P + Qi$

c) Hence, find trigonometric expressions for P and Q

1F nth Roots of Complex Numbers

1.

a) Solve the equation $z^3 = 1$ and represent your solutions on an Argand diagram.



b) Show that the three cube roots of 1 can be written as $1 + \omega + \omega^2$ where $1 + \omega + \omega^2 = 0$

Summary notes:

2. Solve the equation $z^4 - 2\sqrt{3}i = 2$

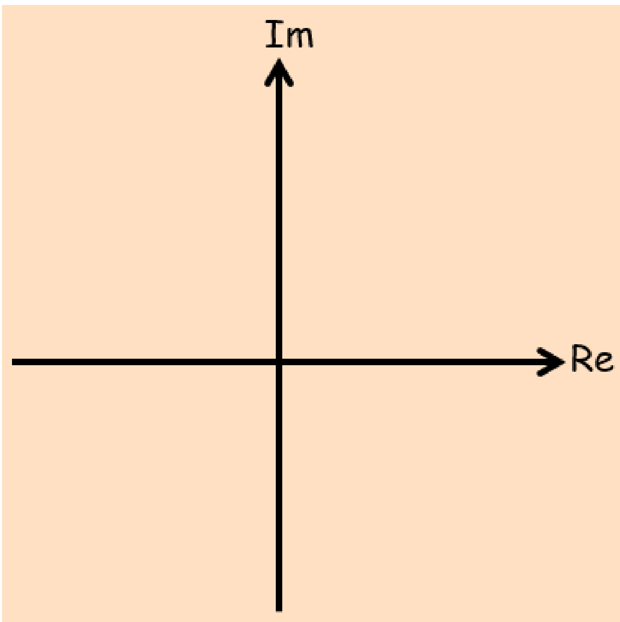
Give your answers in both the modulus-argument and exponential forms.

3. Solve the equation:

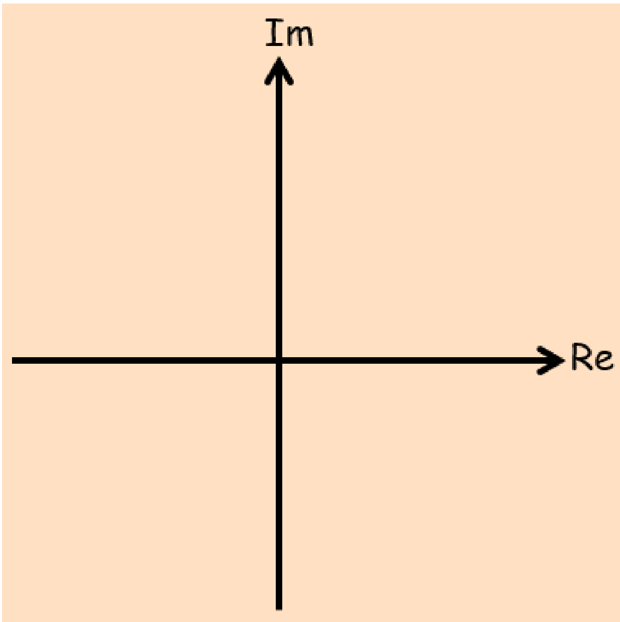
$$z^3 + 4\sqrt{2} + 4i\sqrt{2} = 0$$

1G Geometric Problems with Complex Numbers

1. Find the 6th roots of the complex number $7 + 24i$



2. Find the 6^{th} roots of unity



Summary notes:

3. The coordinate $(\sqrt{3}, 1)$ lies at one vertex of an equilateral triangle. The centre of the triangle is at the origin.

Find the coordinates of the other vertices of the triangle