1) Complex numbers

1.1) Exponential form of complex numbers

Chapter CONTENTS

Worked example	Your turn
Express in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$:	Express in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$:
$3 - 2i$	$2 - 3i$
	$\sqrt{13}e^{-0.983i}$
1	-1
	$e^{i\pi}$
$-i$	i
	$e^{\frac{i\pi}{2}}$

Worked example	Your turn
Express in the form $re^{i\theta}$, where $-\pi < \theta \le \pi$:	Express in the form $re^{i\theta}$, where $-\pi < \theta \le \pi$:
$\sqrt{2}\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$	$\sqrt{3}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$ $\sqrt{3}e^{\frac{\pi i}{8}}$
$\sqrt{13}(\cos(0.983) + i\sin(0.983))$	$\sqrt{13}(\cos(-0.983) + i\sin(-0.983))$ $\sqrt{13}e^{-0.983i}$
$\cos\pi + i\sin\pi$	$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ $e^{\frac{i\pi}{2}}$

Worked example	Your turn
Express in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$:	Express in the form $re^{i\theta}$, where $-\pi < \theta \leq \pi$:
$\sqrt{2}\left(\cos\frac{\pi}{5} - i\sin\frac{\pi}{5}\right)$	$\sqrt{5}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$
	$\sqrt{5}e^{\frac{-\pi i}{3}}$
$\sqrt{3}\left(\cos\left(-\frac{\pi}{7}\right) - i\sin\left(-\frac{\pi}{7}\right)\right)$	

Worked example	Your turn
Express in the form $x + iy$, where $x, y \in \mathbb{R}$:	Express in the form $x + iy$, where $x, y \in \mathbb{R}$:
$\sqrt{2}e^{\frac{3\pi i}{4}}$	$\sqrt{2}e^{-\frac{3\pi i}{4}}$
	$z = -1 - i$
$e^{-\frac{i\pi}{2}}$	$e^{i(0)}$
	$z = 1$

Worked example	Your turn
Express in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$:	Express in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$:
$$2e^{\frac{23\pi i}{5}}$$	$$4e^{\frac{33\pi i}{7}}$$
	$$4(\cos\frac{5\pi}{7} + i\sin\frac{5\pi}{7})$$
$$3e^{\frac{-25\pi i}{8}}$$	

Worked example	Your turn
Use $e^{i\theta} = \cos\theta + i\sin\theta$ to show that $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$	Use $e^{i\theta} = \cos\theta + i\sin\theta$ to show that $\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$$

$$= \cos\theta - i\sin\theta$$

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

1.2) Multiplying and dividing complex numbers

Chapter CONTENTS

Worked example	Your turn
$2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\times 5\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$	$3\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)\times 4\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$ $12i$

Worked example	Your turn
$2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\times 5\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$	$3\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)\times 4\left(\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}\right)$ $6+6i\sqrt{3}$

Worked example	Your turn
$$\frac{2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}{5\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$$	$$\frac{3\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)}{4\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)}$$ $$\frac{3}{8}+\frac{3\sqrt{3}}{8}i$$

Worked example	Your turn
Express in the form $x + iy$ $3e^{\frac{\pi i}{12}} \times \sqrt{2}e^{\frac{\pi i}{4}}$ $5e^{\frac{\pi i}{4}} \times \sqrt{7}e^{\frac{\pi i}{2}}$	Express in the form $x + iy$ $2e^{\frac{\pi i}{6}} \times \sqrt{3}e^{\frac{\pi i}{3}}$ $2i\sqrt{3}$

Worked example	Your turn
Express in the form $x + iy$ $$\frac{3e^{\frac{\pi i}{4}}}{6e^{\frac{\pi i}{12}}}$$ $$\frac{5e^{\frac{\pi i}{4}}}{\sqrt{7}e^{-\frac{\pi i}{2}}}$$	Express in the form $x + iy$ $$\frac{\sqrt{5}e^{-\frac{\pi i}{4}}}{7e^{\frac{\pi i}{2}}}$$ $$-\frac{\sqrt{10}}{14}-\frac{\sqrt{10}}{14}i$$

Worked example	Your turn

Worked example

$$z = 1 - i$$

Find

(a) $|z|$

(b) $\arg(z)$ in terms of π

$$w = 3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$

(d) $\arg\left|\frac{w}{z}\right|$

Your turn

$$z = 5\sqrt{3} - 5i$$

Find

(a) $|z|$ $\quad 10$

(b) $\arg(z)$ in terms of π $\quad -\frac{\pi}{6}$

$$w = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$ $\quad \frac{1}{5}$

(d) $\arg\left|\frac{w}{z}\right|$ $\quad \frac{5\pi}{12}$

Worked example	Your turn
$z = 3 + 3i$ $Im(zw) = 0$ $\|zw\| = 2\|z\|$ Use geometrical reasoning to find the two possibilities for w, giving your answers in exponential form	$z = 2 + 2i$ $Im(zw) = 0$ $\|zw\| = 3\|z\|$ Use geometrical reasoning to find the two possibilities for w, giving your answers in exponential form $w_1 = 3e^{-\frac{\pi i}{4}}, w_2 = 3e^{\frac{3\pi i}{4}}$

1.3) de Moivre's theorem

Worked example	Your turn
Use de Moivre's theorem to express in the form $x + iy$, where $x, y \in \mathbb{R}$	Use de Moivre's theorem to express in the form $x + iy$, where $x, y \in \mathbb{R}$
$(\cos\theta + i\sin\theta)^5$	$(\cos\theta + i\sin\theta)^7$ $\cos 7\theta + i\sin 7\theta$
$(\cos 2\theta + i\sin 2\theta)^3$	$(\cos 3\theta + i\sin 3\theta)^5$ $\cos 15\theta + i\sin 15\theta$

Worked example	Your turn
Express in the form $e^{ni\theta}$	Express in the form $e^{ni\theta}$
$$\frac{(\cos 5\theta + i\sin 5\theta)^3}{(\cos 3\theta + i\sin 3\theta)^7}$$	$$\frac{(\cos 3\theta + i\sin 3\theta)^7}{(\cos 5\theta + i\sin 5\theta)^4}$$
	$e^{i\theta}$
$$\frac{(\cos 2\theta + i\sin 2\theta)^5}{(\cos 7\theta - i\sin 7\theta)^3}$$	

Worked example	Your turn
Simplify $$\frac{\left(\cos\frac{3\pi}{11}+i\sin\frac{3\pi}{11}\right)^2}{\left(\cos\frac{2\pi}{11}-i\sin\frac{2\pi}{11}\right)^{19}}$$	Simplify $$\frac{\left(\cos\frac{9\pi}{17}+i\sin\frac{9\pi}{17}\right)^5}{\left(\cos\frac{2\pi}{17}-i\sin\frac{2\pi}{17}\right)^{3}}$$ $$-1$$

Worked example	Your turn
Express in the form $x + iy$ where $x, y \in \mathbb{R}$ $$(1-\sqrt{3}i)^6$$	Express in the form $x + iy$ where $x, y \in \mathbb{R}$ $$(1+\sqrt{3}\,i)^7$$ $$64 + 64\sqrt{3}\,i$$

Worked example	Your turn
$w = \sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$, find the exact value of: w^6	$z = \sqrt{5}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ Giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$, find the exact value of: z^4 $-\frac{25}{2} + \frac{25\sqrt{3}}{2}i$

Worked example	Your turn
$w = -2 - 2\sqrt{3}i$	$z = -8 + 8\sqrt{3}i$
Using de Moivre's Theorem, find	Using de Moivre's Theorem, find
w^5	z^3
	4096
w^4	

Worked example	Your turn
Use de Moivre's theorem to show that $(a+bi)^n-(a-bi)^n$ is imaginary for all integers n	Use de Moivre's theorem to show that $(a+bi)^n+(a-bi)^n$ is real for all integers n

$$(a+bi)^n+(a-bi)^n$$
$$=\left(re^{i\theta}\right)^n-\left(re^{-i\theta}\right)^n$$
$$=re^{in\theta}-re^{-in\theta}$$
$$=r^n\left(e^{in\theta}-e^{-in\theta}\right)$$
$$=r^n(\cos n\theta+i\sin n\theta-(\cos n\theta-i\sin n\theta)$$
$$=r^n(2i\sin n\theta)$$
$$=2r^n\sin n\theta\,(i)$$

Worked example	Your turn
Using Euler's relation, prove that if n is a positive integer, $\left(r(\cos\theta + i\sin\theta)\right)^n = r^n(\cos n\theta + i\sin n\theta)$	Using Euler's relation, prove that if n is a positive integer, $\left(r(\cos\theta + i\sin\theta)\right)^{-n} = r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$

$$\left(r(\cos\theta + i\sin\theta)\right)^{-n}$$

$$= (re^{i\theta})^{-n}$$

$$= r^{-n}e^{-in\theta}$$

$$= r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$

Worked example	Your turn
Without using Euler's relation, prove that if n is a positive integer, $$\left(r(\cos\theta + i\sin\theta)\right)^n = r^n(\cos n\theta + i\sin n\theta)$$	Without using Euler's relation, prove that if n is a positive integer, $$\left(r(\cos\theta + i\sin\theta)\right)^{-n} = r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$

$$\left(r(\cos\theta + i\sin\theta)\right)^{-n}$$

$$= \frac{1}{\left(r(\cos\theta + i\sin\theta)\right)^n}$$

$$= \frac{1}{r^n(\cos n\theta + i\sin n\theta)}$$

$$= \frac{1}{r^n(\cos n\theta + i\sin n\theta)} \times \frac{\cos n\theta - i\sin n\theta}{\cos n\theta - i\sin n\theta}$$

$$= \frac{\cos n\theta - i\sin n\theta}{r^n(\cos^2 n\theta - i^2\sin^2 n\theta)}$$

$$= \frac{\cos n\theta - i\sin n\theta}{r^n(\cos^2 n\theta + \sin^2 n\theta)}$$

$$= r^{-n}(\cos n\theta - i\sin n\theta)$$

$$= r^{-n}(\cos(-n\theta) + i\sin(-n\theta))$$

1.4) Trigonometric identities

Chapter CONTENTS

Worked example

Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

Your turn

Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Shown

Worked example	Your turn
Use de Moivre's theorem to show that $$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$	Use de Moivre's theorem to show that $$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$$ Shown

Worked example	Your turn
Use de Moivre's theorem to show that $\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$	Use de Moivre's theorem to show that $\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$ Shown

Worked example	Your turn
Use de Moivre's theorem to show that $$\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$$	Use de Moivre's theorem to show that $$\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$$ Shown

Worked example	Your turn
Use de Moivre's theorem to show that $$\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$$ Hence find the exact value of $$\int_0^{\frac{\pi}{2}} \sin^5\theta$$	Use de Moivre's theorem to show that $$\sin^4\theta = \frac{1}{8}\cos 4\theta - \frac{1}{2}\cos 2\theta + \frac{3}{8}$$ Hence find the exact value of $$\int_0^{\frac{\pi}{2}} \sin^4\theta$$ $$\frac{3\pi}{16}$$

Worked example	Your turn
Show that $32\cos^2\theta\sin^4\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$	Show that $32\sin^2\theta\cos^4\theta = -\cos 6\theta - 2\cos 4\theta + \cos 2\theta + 2$ Shown

Worked example	Your turn
Show that $32\cos^2\theta\sin^4\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$ Hence, find the exact value of $\int_0^{\frac{\pi}{3}} \cos^2\theta\sin^4\theta$	Show that $32\sin^2\theta\cos^4\theta = -\cos 6\theta - 2\cos 4\theta + \cos 2\theta + 2$ Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sin^2\theta\cos^4\theta$ $\frac{\pi}{64} + \frac{1}{48}$

Worked example

Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

Hence find the distinct solutions of the equation $16x^5 - 20x^3 + 5x - \frac{1}{2} = 0$, giving your answers to 3 decimal places where necessary.

Your turn

Use de Moivre's theorem to show that

$$\cos 6\theta = 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1$$

Hence find the six distinct solutions of the equation $32x^6 - 48x^4 + 18x^2 - \frac{3}{2} = 0$, giving your answers to 3 decimal places where necessary.

$$x = \pm 0.342, \pm 0.643, \pm 0.985$$

Worked example	Your turn
Use de Moivre's theorem to show that $$\cos 5\theta = \cos\theta\,(16\cos^4\theta - 20\cos^2\theta + 5)$$ Hence solve for $0 \le \theta < \pi$ $$\cos 5\theta - \cos\theta\cos 2\theta = 0$$	Use de Moivre's theorem to show that $$\sin 5\theta = \sin\theta\,(16\cos^4\theta - 12\cos^2\theta + 1)$$ Hence solve for $0 \le \theta < \pi$ $$\sin 5\theta + \cos\theta\sin 2\theta = 0$$ $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}$, 1.209 (3 dp) and 1.932 (3 dp)

1.5) Sums of series

Chapter CONTENTS

Worked example	Your turn
$$\frac{5}{e^{2i\theta}-1}$$	$$\frac{3e^{i\theta}}{e^{4i\theta}-1}$$ $$\frac{3e^{-i\theta}}{2i\sin 2\theta}$$
$$\frac{2e^{i\theta}}{e^{6i\theta}-1}$$	$$\frac{11}{e^{\frac{i\theta}{3}}-1}$$ $$\frac{11e^{-\frac{i\theta}{6}}}{2i\sin\frac{\theta}{6}}$$
$$\frac{7}{e^{\frac{i\theta}{5}}-1}$$	

Worked example	Your turn
$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \cdots + e^{6i\theta}$, for $\theta \neq 2n\pi$, where n is an integer. a) Show that $S = \dfrac{e^{\frac{7i\theta}{2}} \sin 3\theta}{\sin\frac{\theta}{2}}$	$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \cdots + e^{8i\theta}$, for $\theta \neq 2n\pi$, where n is an integer. a) Show that $S = \dfrac{e^{\frac{9i\theta}{2}} \sin 4\theta}{\sin\frac{\theta}{2}}$ Shown

Worked example	Your turn
$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \cdots + e^{6i\theta}$, for $\theta \neq 2n\pi$, where n is an integer.	$S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \cdots + e^{8i\theta}$, for $\theta \neq 2n\pi$, where n is an integer.
a) Show that $S = \frac{e^{\frac{7i\theta}{2}} \sin 3\theta}{\sin\frac{\theta}{2}}$	a) Show that $S = \frac{e^{\frac{9i\theta}{2}} \sin 4\theta}{\sin\frac{\theta}{2}}$
Let $P = \cos\theta + \cos 2\theta + \cos 3\theta + \cdots + \cos 6\theta$ and $Q = \sin\theta + \sin 2\theta + \cdots + \sin 6\theta$	Let $P = \cos\theta + \cos 2\theta + \cos 3\theta + \cdots + \cos 8\theta$ and $Q = \sin\theta + \sin 2\theta + \cdots + \sin 8\theta$
(b) Use your answer to part **a** to show that $P = \cos\frac{7\theta}{2}\sin 3\theta \;\; cosec\,\frac{\theta}{2}$ and find similar expressions for Q and $\frac{Q}{P}$	(b) Use your answer to part **a** to show that $P = \cos\frac{9\theta}{2}\sin 4\theta \;\; cosec\,\frac{\theta}{2}$ and find similar expressions for Q and $\frac{Q}{P}$
	Shown $Q = \sin\frac{9\theta}{2}\sin 4\theta \;\; cosec\,\frac{\theta}{2}$ $\frac{Q}{P} = \tan\frac{9\theta}{2}$

Worked example	Your turn
The convergent infinite series C and S are defined as $$C = 1 + \frac{1}{5}\cos\theta + \frac{1}{25}\cos 2\theta + \frac{1}{125}\cos 3\theta + \cdots$$ $$S = \frac{1}{5}\sin\theta + \frac{1}{25}\sin 2\theta + \frac{1}{125}\sin 3\theta + \cdots$$ a) Find an expression for $C + iS$ b) Hence find an expression for C and S	The convergent infinite series C and S are defined as $$C = 1 + \frac{1}{3}\cos\theta + \frac{1}{9}\cos 2\theta + \frac{1}{27}\cos 3\theta + \cdots$$ $$S = \frac{1}{3}\sin\theta + \frac{1}{9}\sin 2\theta + \frac{1}{27}\sin 3\theta + \cdots$$ a) Find an expression for $C + iS$ b) Hence find an expression for C and S a) $C + iS = \frac{3}{3-e^{i\theta}}$ b) $C = \frac{9-3\cos\theta}{10-6\cos\theta}$ $S = \frac{3\sin\theta}{10-6\cos\theta}$

Worked example	Your turn
The convergent infinite series C and S are defined as $$C = 1 - \frac{1}{3}\cos\theta + \frac{1}{9}\cos 2\theta - \frac{1}{27}\cos 3\theta + \cdots$$ $$S = \frac{1}{3}\sin\theta - \frac{1}{9}\sin 2\theta + \frac{1}{27}\sin 3\theta + \cdots$$ By considering $C - iS$, show that $C = \frac{9+3\cos\theta}{10+6\cos\theta}$ and write down the corresponding expression for S	The convergent infinite series C and S are defined as $$C = 1 - \frac{1}{2}\cos\theta + \frac{1}{4}\cos 2\theta - \frac{1}{8}\cos 3\theta + \cdots$$ $$S = \frac{1}{2}\sin\theta - \frac{1}{4}\sin 2\theta + \frac{1}{8}\sin 3\theta + \cdots$$ By considering $C - iS$, show that $C = \frac{4+2\cos\theta}{5+4\cos\theta}$ and write down the corresponding expression for S Shown $$S = \frac{2\sin\theta}{5 + 4\cos\theta}$$

1.6) nth roots of a complex number

Chapter CONTENTS

Worked example	Your turn
Solve $$z^8 = 1$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $$z^4 = 1$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = i$ $z_3 = -1$ $z_4 = -i$

Worked example	Your turn
Solve $$z^7 - 1 = 0$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $$z^5 - 1 = 0$$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = 0.309 + 0.951i$ $z_3 = -0.809 + 0.588i$ $z_4 = -0.809 - 0.588i$ $z_5 = 0.309 - 0.951i$

Worked example	Your turn
Solve $z^3 = -1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$	Solve $z^3 = 1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ $z_1 = 1$ $z_2 = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $z_3 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Worked example	Your turn
Find the cubic roots of unity, and the value of their sum.	Find the quintic roots of unity, and the value of their sum.

$1, e^{\frac{2\pi}{5}i}, e^{\frac{4\pi}{5}i}, e^{\frac{-4\pi}{5}i}, e^{\frac{-2\pi}{5}i}$

$$z^5 - 1 = 0$$
$$(z-1)(z^4 + z^3 + z^2 + z + 1) = 0$$

$z = e^{\frac{2\pi}{5}i}$, so $z - 1 \neq 0$

$\therefore z^4 + z^3 + z^2 + z + 1 = 0$

Worked example	Your turn
Solve $z^3 = -1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ Represent your solutions on an Argand diagram	Solve $z^3 = 1$ Express the roots in the form $x + iy$, where $x, y \in \mathbb{R}$ Represent your solutions on an Argand diagram

Worked example	Your turn
Solve $z^4 = -2 + 2\sqrt{3}\, i$ Express the roots in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$	Solve $z^4 = 2 + 2\sqrt{3}\, i$ Express the roots in the form $r(\cos\theta + i\sin\theta)$, where $-\pi < \theta \leq \pi$ $z_1 = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ $z_2 = \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ $z_3 = \sqrt{2}\left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right)$ $z_4 = \sqrt{2}\left(\cos\left(-\frac{11\pi}{12}\right) + i\sin\left(-\frac{11\pi}{12}\right)\right)$

Worked example	Your turn
Solve $z^3 + 32\sqrt{2} + 32i\sqrt{2} = 0$ Express the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$	Solve $z^3 + 4\sqrt{2} + 4i\sqrt{2} = 0$ Express the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$ $z_1 = 2e^{-\frac{\pi i}{4}}$ $z_2 = 2e^{\frac{5\pi i}{12}}$ $z_3 = 2e^{-\frac{11\pi i}{12}}$

Worked example	Your turn
Find the three roots of the equation $(z-1)^3=-1$ Plot the points representing these three roots on an Argand diagram. Given that these three points lie on a circle, find its centre and radius	Find the three roots of the equation $(z+1)^3=-1$ Plot the points representing these three roots on an Argand diagram. Given that these three points lie on a circle, find its centre and radius Centre (1,0) ; Radius 1

1.7) Solving geometric problems

Chapter CONTENTS

Worked example	Your turn
The point $P\left(\sqrt{3},-1\right)$ lies at one vertex of an equilateral triangle. The centre of the triangle is at the origin. (a) Find the coordinates of the other vertices of the triangle. (b) Find the area of the triangle.	The point $P\left(\sqrt{3},1\right)$ lies at one vertex of an equilateral triangle. The centre of the triangle is at the origin. (a) Find the coordinates of the other vertices of the triangle. (b) Find the area of the triangle. a) $(-\sqrt{3},1)$ and $(0,-2)$ b) $3\sqrt{3}$

Worked example	Your turn
The point $P(1, -\sqrt{3})$ lies at one vertex of a regular pentagon. The centre of the polygon is at the origin. Find the coordinates of the other vertices.	The point $P(-1, \sqrt{3})$ lies at one vertex of a regular pentagon. The centre of the polygon is at the origin. Find the coordinates of the other vertices. Round your answers to 2 decimal places. $(-1.96, 0.42)$ $(-0.21, -1.99)$ $(1.83, -0.81)$ $(1.34, 1.49)$