**Paper 1: Core Pure Mathematics 1 Mark Scheme**

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **1** |  | M1 | 3.1a |
|  | M1 | 2.1 |
|  | A1 | 2.2a |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(5)** |  |
|  | **Alternative by induction:** | M1 | 3.1a |
| Assume true for *n* = *k*  so |  |  |
|  | M1 | 2.1 |
|  | A1 | 2.2a |
|  | M1 | 1.1b |
| So true for *n* = *k* + 1  So | A1 | 1.1b |
|  | **(5)** |  |
| **(5 marks)** | | | |

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| **Question 1 notes:** |
| **Main Scheme**  **M1:** Valid attempt at partial fractions  **M1:** Starts the process of differences to identify the relevant fractions at the start and end  **A1:** Correct fractions that do not cancel  **M1:** Attempt common denominator  **A1:** Correct answer |
| **Alternative by Induction:**  **M1:** Uses *n* = 1 and *n* = 2 to identify values for *a* and *b*  **M1:** Starts the induction process by adding the (*k* + 1)th term to the sum of *k* terms  **A1:** Correct single fraction  **M1:** Attempt to factorise the numerator  **A1:** Correct answer and conclusion |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **2** | When *n* = 1,  391 = 17 × 23 so the statement is true for *n* = 1 | B1 | 2.2a |
| Assume true for *n* = *k* so is divisible by 17 | M1 | 2.4 |
|  | M1 | 2.1 |
|  |  |  |
|  | A1 | 1.1b |
|  | A1 | 1.1b |
| If the statement is true for *n* = *k* then it has been shown true for  *n* = *k* + 1 and as it is true for *n* = 1, the statement is true for all positive integers *n* | A1 | 2.4 |
|  | **(6)** |  |
| **(6 marks)** | | | |
| **Notes:** | | | |
| **B1:** Shows the statement is true for *n* = 1  **M1:** Assumes the statement is true for *n* = *k*  **M1:** Attempts f(*k*+1) – f(*k*)  **A1:** Correct expression in terms of f(*k*)  **A1:** Correct expression in terms of f(*k*)  **A1:** Obtains a correct expression for f(*k* + 1)  **A1:** Correct complete conclusion | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **3** | is also a root | B1 | 1.2 |
| **or**  Sum of roots = 6, Product of roots = 13 | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1a |
|  | A1 | 1.1b |
| Im  (-1, 2)  (3, 2)  Re  (-1, -2)  (3, -2) | B1  3 ± 2i  Plotted  correctly | 1.1b |
| B1ft  ̶ 1 ± 2i  Plotted  correctly | 1.1b |
| **(9 marks)** | | | |
| **Notes:** | | | |
| **B1:** Identifies the complex conjugate as another root  **M1:** Uses the conjugate pair and a correct method to find a quadratic factor  **A1:** Correct quadratic  **M1:** Uses the given quartic and their quadratic to identify the value of *c*  **A1:** Correct 3TQ  **M1:** Solves their second quadratic  **A1:** Correct second conjugate pair  **B1:** First conjugate pair plotted correctly and labelled  **B1ft:** Second conjugate pair plotted correctly and labelled (Follow through their second  conjugate pair) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **4** |  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
| Area of triangle = | M1 | 3.1a |
| Area of *R* = | M1 | 1.1b |
|  | A1 | 1.1b |
| **(9 marks)** | | | |
| **Notes:** | | | |
| **M1:** Realises the angle for *A* is required and attempts to find it  **A1:** Correct angle  **M1:** Uses a correct area formula and squares *r* to achieve a 3TQ integrand in cos 2*θ*  **M1:** Use of the correct double angle identity on the integrand to achieve a suitable form for  integration  **A1:** Correct integration  **M1:** Correct use of limits  **M1:** Identifies the need to subtract the area of a triangle and so finds the area of the triangle  **M1:** Complete method for the area of *R*  **A1:** Correct final answer | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **5(a)** | Pond contains 1000 + 5*t* litres after *t* days | M1 | 3.3 |
| If *x* is the amount of pollutant in the pond after *t* days  Rate of pollutant out = g per day | M1 | 3.3 |
| Rate of pollutant in = 25  2 g = 50g per day | B1 | 2.2a |
|  | A1\* | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 3.1b |
|  | A1 | 1.1b |
|  | M1 | 3.4 |
|  | M1 | 1.1b |
| = 370g | A1 | 2.2b |
|  | **(5)** |  |
| **(c)** | e.g.   * The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry * The rate of leaking could be made to vary with the volume of water in the pond | B1 | 3.5c |
|  | **(1)** |  |
| **(10 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Forms an expression of the form 1000 + *kt* for the volume of water in the pond at time *t*  **M1:** Expresses the amount of pollutant out in terms of *x* and *t*  **B1:** Correct interpretation for pollutant entering the pond  **A1\*:** Puts all the components together to form the correct differential equation | | | |
| **(b)**  **M1:** Uses the model to find the integrating factor and attempts solution of their differential  equation  **A1:** Correct solution  **M1:** Interprets the initial conditions to find the constant of integration  **M1:** Uses their solution to the problem to find the amount of pollutant after 8 days  **A1:** Correct number of grams | | | |
| **(c)**  **B1:** Suggests a suitable refinement to the model | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **6(a)** |  | B1 | 3.1a |
|  | M1 | 1.1b |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 1.1b |
| Mean value = | M1 | 2.1 |
|  | A1\* | 2.2a |
|  | **(3)** |  |
| **(c)** |  | M1 | 2.2a |
|  | A1 | 1.1b |
|  | **(2)** |  |
| **(9 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **B1:** Splits the fraction into two correct separate expressions  **M1:** Recognises the required form for the first integration  **M1:** Recognises the required form for the second integration  **A1:** Both expressions integrated correctly and added together with constant of integration  included | | | |
| **(b)**  **M1:** Uses limits correctly and combines logarithmic terms  **M1:** Correctly applies the method for the mean value for their integration  **A1\*:** Correct work leading to the given answer | | | |
| **(c)**  **M1:** Realises thatthe effect of the transformation is to increase the mean value by ln *k*  **A1:** Combines ln’s correctly to obtain the correct expression | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **7(a)** |  | M1 | 2.1 |
|  | M1 | 2.1 |
|  | M1 | 2.1 |
|  | A1\* | 1.1b |
|  | **(4)** |  |
| **(b)** |  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | M1 | 3.4 |
| **or** awrt 5.03 | A1 | 1.1b |
|  | **(4)** |  |
| **(8 marks)** | | | |
| **Notes:** | | | |
| **(a)**  **M1:** Obtains *x* in terms of *y* and cos *θ*  **M1:** Obtains an equation connecting *y* and sin *θ*  **M1:** Uses Pythagoras to obtain an equation in *x* and *y* only  **A1\*:** Obtains printed answer | | | |
| **(b)**  **M1:** Uses the correct volume of revolution formula with the given expression  **A1:** Correct integration  **M1:** Correct use of correct limits  **A1:** Correct volume | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **8** |  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
|  | M1 | 3.1a |
|  | A1 | 1.1b |
|  | M1 | 3.1a |
| e.g. | A1 | 2.5 |
|  | **(7)** |  |
| **(7 marks)** | | | |
| **Notes:** | | | |
| **M1:** Substitutes the parametric equation of the line into the equation of the plane and solves for  *λ*  **A1:** Obtains the correct coordinates of the intersection of the line and the plane  **M1:** Substitutes the parametric form of the line perpendicular to the plane passing through  (2, 4, 6) into the equation of the plane to find *t*  **M1:** Find the reflection of (2, 4, 6) in the plane  **A1:** Correct coordinates  **M1:** Determines the direction of *l* by subtracting the appropriate vectors  **A1:** Correct vector equation using the correct notation  π  (8,8, 0)  (10, 0,4)  (2, 4,6) | | | |

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| **Question** | **Scheme** | **Marks** | **AOs** |
| **9(a)(i)** | Weight = massg  But mass is in thousands of kg, so *m* = 3 | M1 | 3.3 |
| **(ii)** |  | M1 | 1.1b |
|  | M1 | 1.1b |
| = 200 cos *t* so PI is *x* = 40 sin *t* – 20 cos | A1\* | 2.1 |
| **or** |  |  |
| Let | M1 | 1.1b |
|  | M1 | 2.1 |
|  | A1\* | 1.1b |
| **(iii)** |  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | M1 | 1.1b |
|  | A1 | 1.1b |
|  | **(8)** |  |
| **(b)** |  | M1 | 3.4 |
|  | M1 | 3.4 |
|  | A1 | 1.1b |
|  | A1 | 3.4 |
|  | (4) |  |
| **(12 marks)** | | | |

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| **Question 9 notes:** |
| **(a)(i)**  **M1:** Correct explanation that in the model, *m* = 3 |
| **(ii)**  **M1:** Differentiates the given PI twice  **M1:** Substitutes into the given differential equation  **A1\*:** Reaches 200cos*t* and makes a conclusion  **or**  **M1:** Uses the correct form for the PI and differentiates twice  **M1:** Substitutes into the given differential equation and attempts to solve  **A1\*:** Correct PI |
| **(iii)**  **M1:** Uses the model to form and solve the auxiliary equation  **A1:** Correct complementary function  **M1:** Uses the correct notation for the general solution by combining PI and CF  **A1:** Correct General Solution for the model |
| **(b)**  **M1:** Uses the initial conditions of the model, *t* = 0 at *x* = 0, to form an equation in *A* and *B*  **M1:** Uses  0 at *x* = 0 in the model to form an equation in *A* and *B*  **A1:** Correct PS  **A1:** Obtains 33m using the assumptions made in the model |