Write your name here

Surname		Other names

Centre Number

Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Paper 2J: Further Mechanics 1 and Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

SECTION A

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A small ball of mass $0.1\,\mathrm{kg}$ is dropped from a point which is $2.4\,\mathrm{m}$ above a horizontal floor. The ball falls freely under gravity, strikes the floor and bounces to a height of $0.6\,\mathrm{m}$ above the floor. The ball is modelled as a particle.

   (a) Show that the coefficient of restitution between the ball and the floor is 0.5

   (6)

   (b) Find the height reached by the ball above the floor after it bounces on the floor for the second time.

   (3)

   (c) By considering your answer to (b), describe the subsequent motion of the ball.

   (1)

Question 1 continued

(Total for Question 1 is 10 marks)

2. A small stone of mass $0.5\,\text{kg}$ is thrown vertically upwards from a point A with an initial speed of $25\,\text{m\,s}^{-1}$. The stone first comes to instantaneous rest at the point B which is $20\,\text{m}$ vertically above the point A. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from A to B.

(3)

The air resistance is modelled as a constant force of magnitude R newtons.

(b) Find the value of R.

(2)

(c) State how the model for air resistance could be refined to make it more realistic.

(1)

Question 2 continued

(Total for Question 2 is 6 marks)

3. [*In this question use g = 10 m s⁻²*]

A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4 \, \text{m s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin \alpha = \dfrac{1}{15}$. Because of the hill, the jogger reduces her speed to $3 \, \text{m s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3 \, \text{m s}^{-1}$.

(5)

(Total for Question 3 is 8 marks)

4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal table. A particle Q of mass m is moving in the opposite direction to P along the same straight line. The particles collide directly. Immediately before the collision the speed of P is u and the speed of Q is $2u$. The velocities of P and Q immediately after the collision, measured in the direction of motion of P before the collision, are v and w respectively. The coefficient of restitution between P and Q is e.

(a) Find an expression for v in terms of u and e.

(6)

Given that the direction of motion of P is changed by the collision,

(b) find the range of possible values of e.

(2)

(c) Show that $w = \dfrac{u}{4}(1 + 9e)$.

(2)

Following the collision with P, the particle Q then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

Given that $e = \dfrac{5}{9}$, and that P and Q collide again in the subsequent motion,

(d) find the range of possible values of f.

(6)

Question 4 continued

(Total for Question 4 is 16 marks)

TOTAL FOR SECTION A IS 40 MARKS

SECTION B

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8 \, \text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

5. A particle P moves on the x-axis. At time t seconds the velocity of P is $v \, \text{m s}^{-1}$ in the direction of x increasing, where

$$v = (t - 2)(3t - 10), \qquad t \geqslant 0$$

When $t = 0$, P is at the origin O.

(a) Find the acceleration of P at time t seconds.

(2)

(b) Find the total distance travelled by P in the first 2 seconds of its motion.

(3)

(c) Show that P never returns to O, explaining your reasoning.

(3)

Question 5 continued

(Total for Question 5 is 8 marks)

6. A light inextensible string has length $7a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 5a$. A particle of mass m is attached to a point P on the string where $AP = 4a$. The particle moves in a horizontal circle with constant angular speed ω, with both AP and BP taut.

(a) Show that

   (i) the tension in AP is $\dfrac{4m}{25}(9a\omega^2 + 5g)$

   (ii) the tension in BP is $\dfrac{3m}{25}(16a\omega^2 - 5g)$.

                                                     (10)

The string will break if the tension in it reaches a magnitude of $4mg$.

The time for the particle to make one revolution is S.

(b) Show that

$$3\pi\sqrt{\frac{a}{5g}} < S < 8\pi\sqrt{\frac{a}{5g}}$$

                                                     (5)

(c) State how in your calculations you have used the assumption that the string is light.
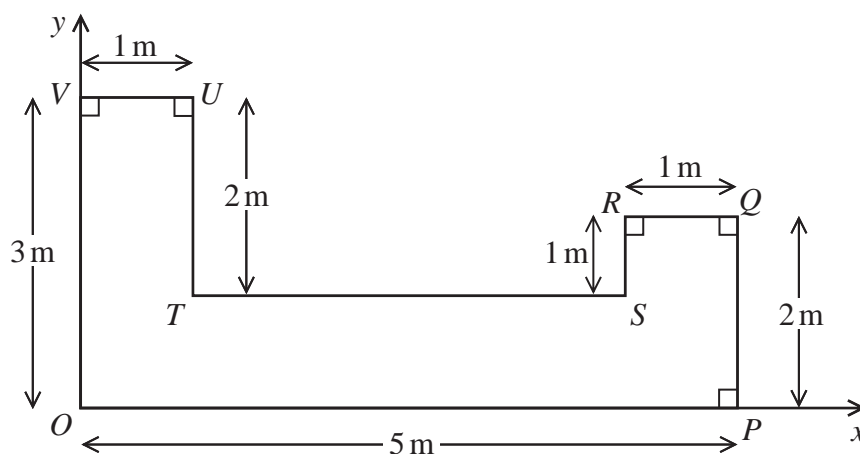
                                                     (1)

Question 6 continued

Question 6 continued

(Total for Question 6 is 16 marks)

7.

Figure 1

Figure 1 shows the shape and dimensions of a template *OPQRSTUV* made from thin uniform metal.

$OP = 5\,\text{m}$, $PQ = 2\,\text{m}$, $QR = 1\,\text{m}$, $RS = 1\,\text{m}$, $TU = 2\,\text{m}$, $UV = 1\,\text{m}$, $VO = 3\,\text{m}$.

Figure 1 also shows a coordinate system with *O* as origin and the *x*-axis and *y*-axis along *OP* and *OV* respectively. The unit of length on both axes is the metre.

The centre of mass of the template has coordinates $(\bar{x}, \bar{y})$.

(a) (i) Show that $\bar{y} = 1$

   (ii) Find the value of $\bar{x}$.

(7)

A new design requires the template to have its centre of mass at the point (2.5, 1). In order to achieve this, two circular discs, each of radius *r* metres, are removed from the template which is shown in Figure 1, to form a new template *L*. The centre of the first disc is (0.5, 0.5) and the centre of the second disc is (0.5, *a*) where *a* is a constant.

(b) Find the value of *r*.

(4)

(c) (i) Explain how symmetry can be used to find the value of *a*.

   (ii) Find the value of *a*.

(2)

The template *L* is now freely suspended from the point *U* and hangs in equilibrium.

(d) Find the size of the angle between the line *TU* and the horizontal.

(3)

(Total for Question 7 is 16 marks)

TOTAL FOR SECTION B IS 40 MARKS
TOTAL FOR PAPER IS 80 MARKS