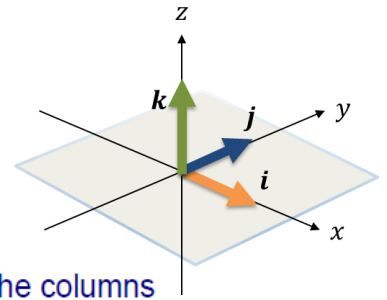


Linear Transformations in 3D

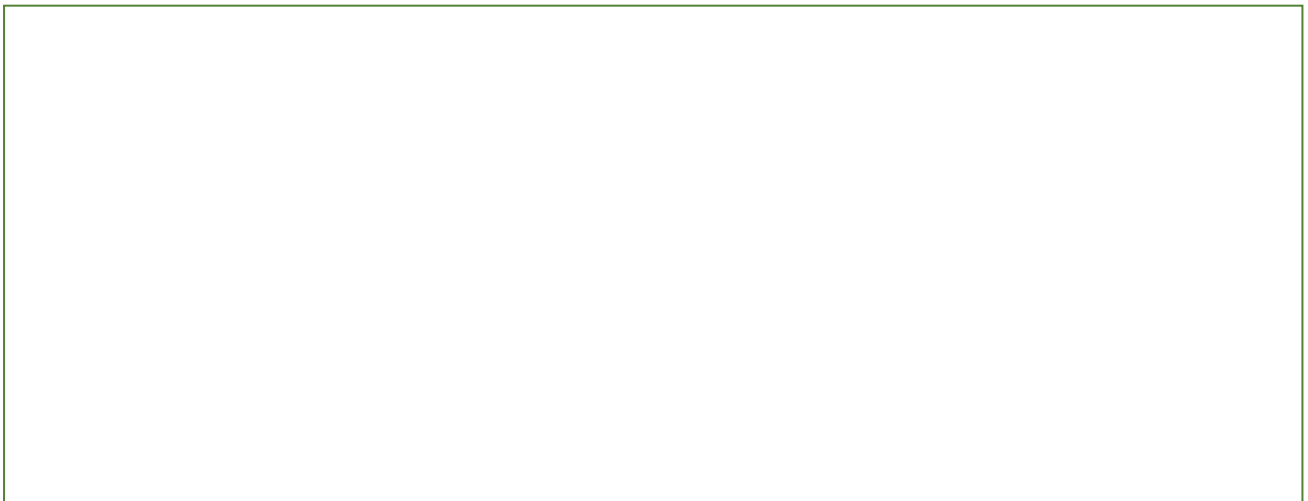
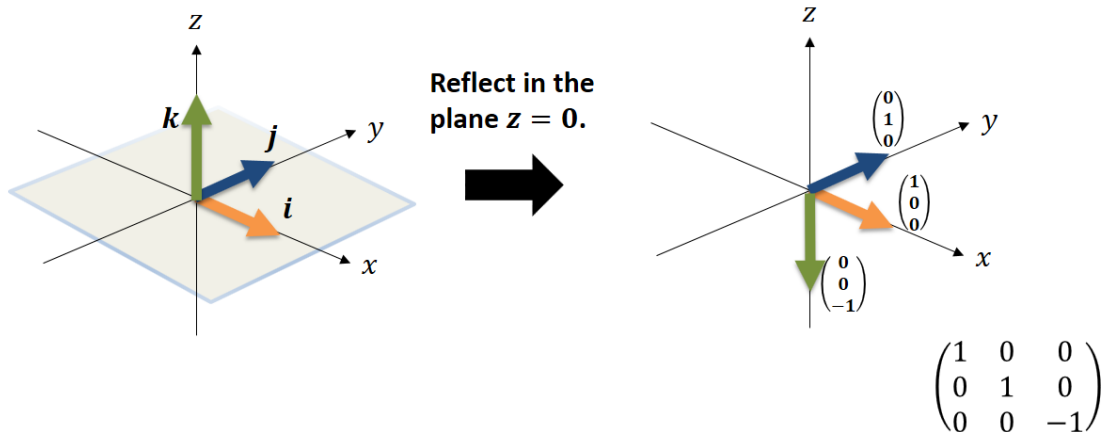


For a transformation in three dimensions, represented by a 3×3 matrix, the columns of the matrix represent the images of the point $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ respectively.

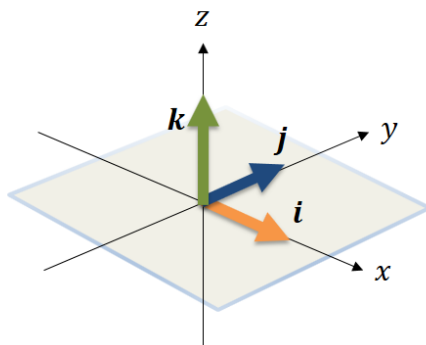
In most of the simple transformations in three dimensions that you will meet, you will see that at least one of the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ maps to itself. One way to identify the transformation is to ignore the row and column for this point, and look at the remaining 2×2 matrix. Identify the transformation, and then express it in terms of a three dimensional transformation.

The list below explains how to recognise each of the different types of three-dimensional transformation that you might meet.

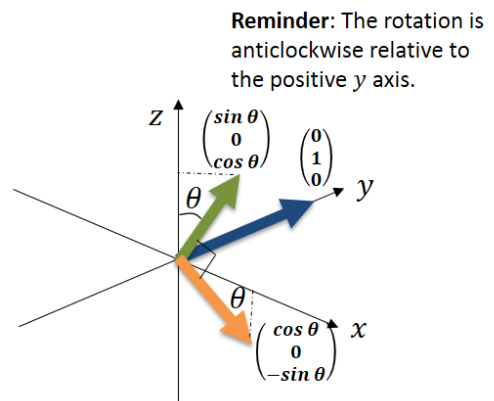
Reflections



Rotations



Rotate by
angle θ about
the y -axis.



Test Your Understanding

$$\mathbf{M} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$$

- (a) Describe the transformation represented by \mathbf{M} .
- (b) Find the image of the point with coordinates $(-1, -2, 1)$ under the transformation represented by \mathbf{M} .