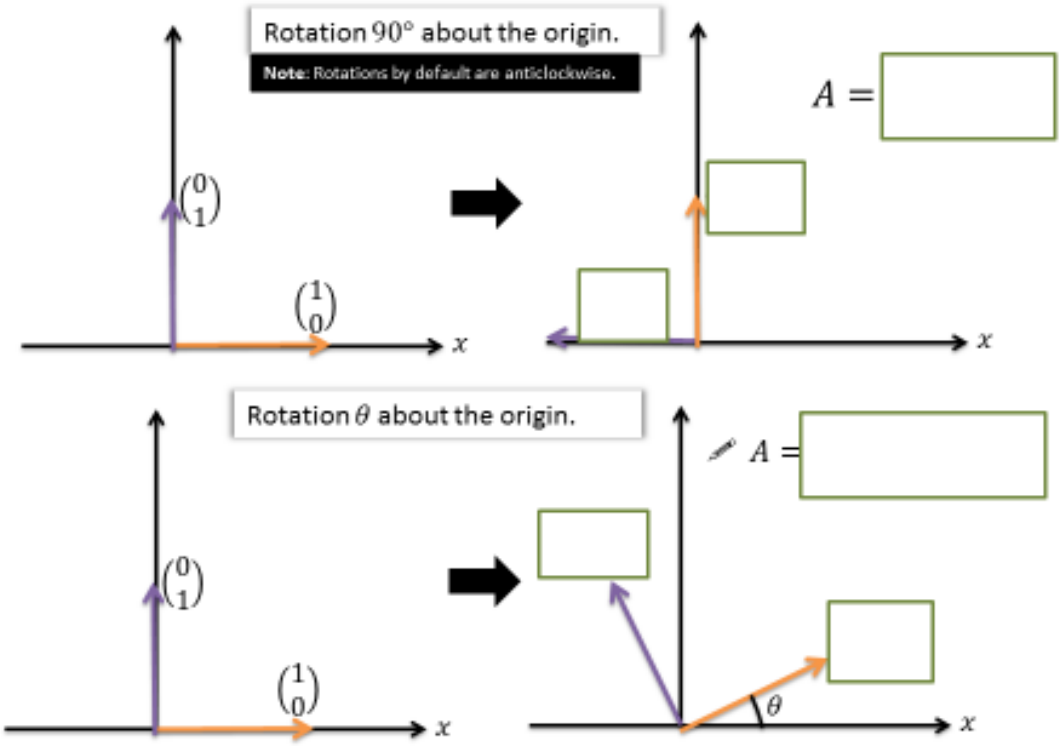
Determining a matrix for a transformation

Rotations



Reflections

Reflection in the x-axis leaves the point (1, 0) unchanged but maps the point (0, 1) to the point (0, -1).

So the matrix representing this transformation is

Reflection in the y-axis maps the point (1, 0) to the point (-1, 0) but leaves the point (0, 1) unchanged.

So the matrix representing this transformation is

Reflection in the line $y = x$ maps the point (1, 0) to the point (0, 1) and maps the point (0, 1) to the point (1, 0).

So the matrix representing this transformation is

Reflection in the line $y = -x$ maps the point (1, 0) to the point (0, -1) and maps the point (0, 1) to the point (-1, 0).

So the matrix representing this transformation is

Warning!! Often mistaken for a rotation!

Test Your Understanding

1. Find the matrix representing a reflection in the line $y = x$.

2. Find the matrix representing a rotation by $270°$.

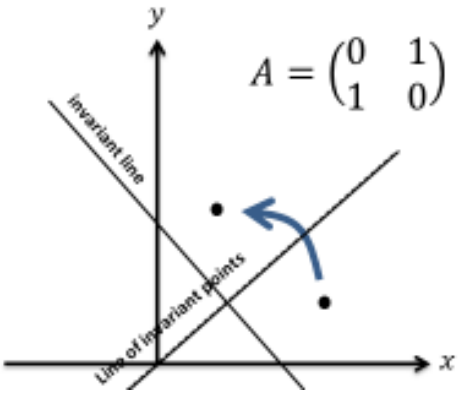
3.

$$\mathbf{C} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the transformations described by matrix $\mathbf{C}$.

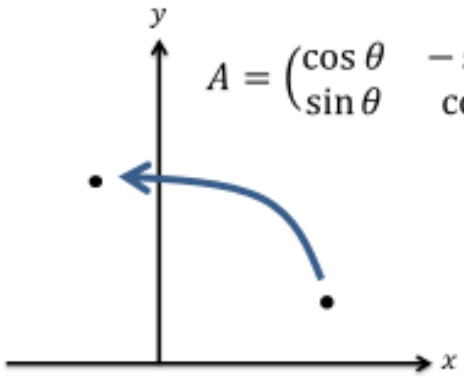
Invariant Lines and Points

Reflection in $y = x$



$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Rotation



$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Finding Invariant Points

Finding Invariant Lines

Example

Find the line of invariant points and invariant lines of the matrix $\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$.

Activity

If possible, fill in either a matrix or a type of transformation (such as reflection or enlargement) that satisfies the conditions for each cell in the grid. If any are not possible, explain why.

		Invariant points	
		Only the origin	Line of invariant points
Invariant lines	none		
	finite number		
	infinite number		